Sample Lessons

Developing Fractions Sense®

Student Workbook C

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Student Name: ____

Teacher Name: ____

Grade: __________________ Year: ___________________________

Lesson 17

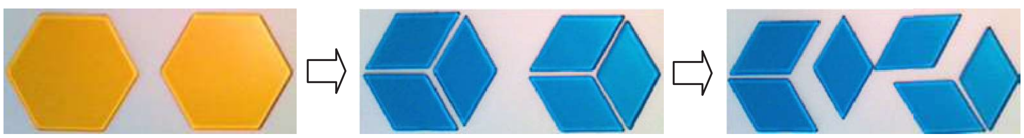
Understanding the Relationship Between $a \div b$ and $\frac{a}{b}$: Part I

Let's consider this problem:

Your Mom would like to share two candy bars of the same size equally among three kids. How much of the candy bar should she give each kid?

One way to solve this problem is to use our fraction blocks, and to represent a whole candy bar by the yellow block. Hence, two candy bars are represented by 2 yellow blocks. Our task now is to break up the 2 yellow blocks into three equal parts.

In order to accomplish this, we will first replace each yellow block by three blue blocks, as shown in the middle image below. Next, we can partition those 6 blue blocks into 3 equal parts of two blue blocks each. Since each blue block represents one-third of a candy bar, we see that each kid should get $\frac{2}{3}$ of a candy bar.



We can use the mathematical notation "$2 \div 3$" to indicate that we need to partition the 2 candy bars into 3 equal parts. Since the size of each of those parts is $\frac{2}{3}$, we have this relationship: $2 \div 3 = \frac{2}{3}$. We read this equation as "Two divided by 3 is two-thirds."

Please use your fraction blocks to answer these sharing problems:

a. One cake shared equally among 2 kids. Each one gets _____of the cake.

b. Two cakes shared equally among 6 kids. Each one gets _____ of a cake.

c. Two cakes shared equally among 4 kids. Each one gets _____ of a cake.

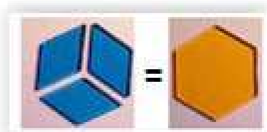
d. $1 \div 2 =$ e. $2 \div 6 = \frac{2}{6} =$ f. $2 \div 4 =$

What we have learned:
A sharing problem can be expressed using division. If we share 2 cakes among 3 kids, each one gets $\frac{2}{3}$ of a cake. We can express this as $2 \div 3 = \frac{2}{3}$.

Lesson 22

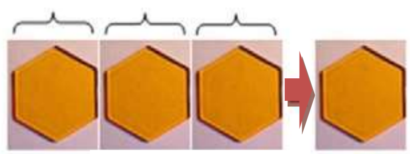
Understanding the Distinction Between $3 \times \frac{1}{3}$ and $\frac{1}{3} \times 3$

We already know that in the expression $3 \times \frac{1}{3}$, the whole number 3 is the multiplier, and the unit fraction $\frac{1}{3}$ is the multiplicand. To calculate the value of $3 \times \frac{1}{3}$, we take three copies of the unit fraction $\frac{1}{3}$. We see below that $3 \times \frac{1}{3} = 1$.



$$3 \times \frac{1}{3} = 1$$

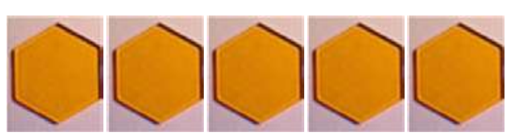
Now, let's consider the meaning of $\frac{1}{3} \times 3$. Here, the unit fraction $\frac{1}{3}$ is the multiplier. Since the whole number 3 is the multiplicand, our task is to take one-third of 3. We will model the problem by beginning with 3 wholes, represented by three yellow blocks. In order to take $\frac{1}{3}$ of these 3 blocks, we will partition them into 3 equal parts, and select one part, namely one yellow block. Hence, $\frac{1}{3} \times 3 = 1$.



$$\frac{1}{3} \times 3 = 1$$

Whereas the products $3 \times \frac{1}{3}$ and $\frac{1}{3} \times 3$ both equal 1, the two expressions have different meaning. When the multiplier is 3, we need to take 3 copies of the multiplicand. When the multiplier is $\frac{1}{3}$, we need to take one-third of the multiplicand.

Use the picture below to determine the value of $\frac{1}{5} \times 5 =$ ________



What we have learned:

The expressions $3 \times \frac{1}{3}$ and $\frac{1}{3} \times 3$ have different meanings. In each expression, the first number is the multiplier and the second is the multiplicand.

Lesson 30

A Visual Model for Fraction Multiplication

In this lesson we show how the symbolic partition of the multiplicand into equal parts can serve as a visual model for fraction multiplication.

Let's consider the product $\frac{3}{4} \times \frac{8}{5}$. We are able to partition $\frac{8}{5}$ into 4 equal parts, $\frac{8}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$. Since one of those 4 equal parts is $\frac{2}{5}$, we have $\frac{1}{4} \times \frac{8}{5} = \frac{2}{5}$.

$$\frac{8}{5} = \overline{\frac{2}{5}} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$$

Since 3 of those parts, $\frac{2}{5} + \frac{2}{5} + \frac{2}{5}$, sum to $\frac{6}{5}$, we have $\frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$.

$$\frac{8}{5} = \overline{\frac{2}{5} + \frac{2}{5} + \frac{2}{5}} + \frac{2}{5}$$

A. Find each product by using the given partition. The first one is done for you.

a. $\frac{1}{3} \times \frac{6}{7}$	$\frac{6}{7} = \overline{\frac{2}{7}} + \frac{2}{7} + \frac{2}{7}$	**Answer:** $\frac{1}{3} \times \frac{6}{7} = \frac{2}{7}$
b. $\frac{2}{3} \times \frac{6}{7}$	$\frac{6}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$	**Answer:** $\frac{2}{3} \times \frac{6}{7} =$
c. $\frac{3}{3} \times \frac{6}{7}$	$\frac{6}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$	**Answer:** $\frac{3}{3} \times \frac{6}{7} =$

B. Find each product by using the given partition.

a. $\frac{1}{3} \times \frac{3}{5}$	$\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$	**Answer:** $\frac{1}{3} \times \frac{3}{5} =$
b. $\frac{2}{3} \times \frac{3}{5}$	$\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$	**Answer:** $\frac{2}{3} \times \frac{3}{5} =$

C. Review Lesson #29. Use your fraction blocks to find these products.

a. $\frac{3}{4} \times \frac{8}{3} =$ **b.** $\frac{2}{3} \times \frac{3}{2} =$ **c.** $\frac{3}{4} \times \frac{8}{6} =$

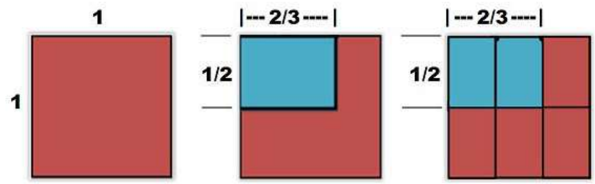
What we have learned:

A partition of a fraction into a sum of equal fractions, such as $\frac{8}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$, enables us to visually see that $\frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$, since 3 of the 4 equal parts sum to $\frac{6}{5}$.

Lesson 37

Finding the Area of a Rectangle with Fractional Side Lengths: Part I

Let's find the area of a rectangle whose sides are $\frac{1}{2}$ and $\frac{2}{3}$.

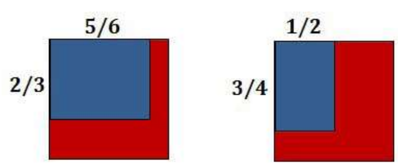


At the left above we have a square with dimensions 1 by 1. Its area is one square unit. In the middle, in blue, we section off a rectangle with dimensions $\frac{2}{3}$ and $\frac{1}{2}$. We need to find the area of this blue portion.

At the right, we draw vertical lines and extend the horizontal line as shown. We see that we have a total of 6 equal small rectangles, which together make up the area of the square. Since the area of the square is 1, each small rectangle has the area $\frac{1}{6}$ square units.

Now, the blue portion consists of two of those rectangles. Hence, its area is $\frac{2}{6}$ square units. We notice that the area of the blue portion, $\frac{2}{6}$, is also the product of the two sides of the blue rectangle, $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$. In general, the area A of a rectangle with fractional sides $\frac{a}{b}$ and $\frac{c}{d}$ will be the product of its sides, e.g., $A = \frac{a}{b} \times \frac{c}{d}$.

The area of each square below is 1 square foot.



a. What is the area of the rectangle with dimensions $\frac{2}{3}$ by $\frac{5}{6}$?

b. What is the area of the rectangle with dimensions $\frac{3}{4}$ by $\frac{1}{2}$?

What we have learned:
The area, A, of a rectangle with fractional sides $\frac{a}{b}$ and $\frac{c}{d}$ will be the product of its sides. Hence, $A = \frac{a \times c}{b \times d}$.