Hands-On Equations® Verbal Problems Research Study: 3rd Grade Gifted Students

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Hands-On Equations Interim Report: May 1, 2009

Abstract

The Hands-On Equations Verbal Problems Book and the Day2 Hands-On Equations Verbal Problems Workshop presents a unique method by which students can represent and solve verbal problems. The essence of the method, as described in more detail in the study, is to transform an abstract process into a visual and kinesthetic one.

The purpose of the current study is to examine the performance of 195 gifted 3rd graders on specific verbal problems via pre- and post-tests. The pre-tests were provided after the students had completed Level I of Hands-On Equations but prior to receiving instruction on how to apply HOE to the solution of verbal problems.

The results suggest that the visual representational methods of HOE can significantly enhance student ability to solve the problems presented in this study.

Introduction:

Hands-On Equations® is a system of instruction which enables grade school students to solve algebraic linear equations with unknowns on both sides of the equation. The system was first introduced in 1986, and since then has been used by many classrooms in the United States. More recently, Henry Borenson, the inventor of Hands-On Equations, developed a means, or system of instruction, to enable grade school and middle school students to solve verbal problems using the Hands-On Equations game pieces and methodology. This instructional approach is noted in the Hands-On Equations Verbal Problems Book and is presented in the Hands-On Equations Verbal Problems Workshops conducted by Borenson and Associates.

The essence of the Hands-On Equations verbal problems approach is to represent the unknown aspects of the problem using physical game pieces or the pictorial notation, rather than the abstract notation normally used in a traditional algebra class. Additionally, the student uses a physical or pictorial setup rather than an abstract equation, and finally, solves the problem using kinesthetic actions, rather than abstract rules of algebra. Below we will illustrate the two approaches with an example from the current study.

Kathy purchased 5 bags of candy for the same amount as Janie, who purchased 3 of the same bags of candy and a $4 bag of potato chips. What was the price of each bag of candies?

Traditional Solution:

Let x be the price of a bag of candies.

Then 5x is the price of five bags of candy

And 3x is the price of three bags of candy

The equation is: $5x = 3x + 4$. So, $x= 2$. The price of each bag of candies is $2.

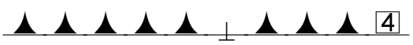
Check: $10 = 10$.

Hands-On Equations Solution:

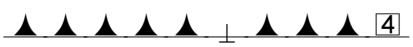
Let be the price of a bag of candies

Then is the cost of 5 boxes of candy

And is the cost of 3 boxes of candy

The setup is,  ; =2. Each box cost $2. Check: 10 = 10.

We notice that both the traditional and the Hands-On Equations approach require the student to represent the unknown elements. In the traditional approach, the abstract notation is used (such as using "5x" to represent the price of five boxes of candy). In the HOE approach, the students represent the cost of the five boxes using the icon for the cost of one box, and they represent the cost of the five boxes as . Hence, the representation of the unknowns in HOE is visual instead of abstract.

Next, the representation of the equation in the traditional notation is 5x = 3x + 4; in HOE it is  . Hence the representation of the equation in HOE is physical or visual, instead of abstract,

Next, the solution method in the traditional notation involves using algebraic rules or principles applied to the abstract equation. In HOE, the solution involves either using physical legal moves, or using arrows to remove the like elements if using the pictorial notation.

Finally, in the traditional notation, the check is carried out in the original problem or in the abstract equation. In HOE, it is carried out in the original problem or in the physical or pictorial notation.

It is known that students have great difficulty with Algebra 1. In one large district in the U.S., 91% of the students failed the first term algebra 1 exam. There are many components to Algebra 1, but solving equations and working with word problems is an integral aspect of Algebra 1. Hence if students can gain facility in working with algebraic linear equations and in working with word problems, they will have an important skill needed for success in Algebra 1.

The current study is expected to be the first of a number of studies to determine if the HOE instructional approach to verbal problems produces significant verbal-problem solving gains (from pre-test to post-test) in students from the 3rd grade through high school. It is hoped that additional studies, will enable us to measure if the experience of the students enable them to transfer the procedure to problems other than those to which they had

been provided with specific instruction. Furthermore, it is hoped that additional studies will afford the opportunity to do a comparison study.

Purpose of Current Study:

The current study, conducted with 3rd grade gifted students, intends to gather pre- and post-test data to ascertain if the HOE instructional approach to verbal problems leads to significant student gains in solving specific verbal problems which require an algebraic solution. All the students and teachers participating in this study are in a suburban district in central Texas.

Overview:

In May 2008, 27 teachers of 3rd grade gifted students were provided with a full-day Hands-On Equations Verbal Problems workshop. In October 2008, these teachers administered a six-question verbal problem pre-test to their 195 3rd grade gifted students. All of the students had completed Level I of Hands-On Equations. The students were then provided with six verbal problems lessons to be solved using the Hands-On Equations approach to word problems. At the conclusion of the six lessons in December 2008, the students were provided with a post-test, which consisted of six verbal problems similar to those provided on the pre-test. A t-test was conducted to determine if there was a significant increase in performance from the pre-test to the post-test.

Rationale:

Traditionally, students have great difficulty in solving verbal problems. The purpose of the present study was to obtain preliminary data of the extent to which the HOE methods of instruction can improve student performance on verbal problems. The scope of the study was narrow in the sense that the purpose was to see the extent to which students could learn the various “problem types” which were presented on the pre-test. No attempt was made in this study to see if students could transfer their learning to solving problems which were distinctly different from those presented and taught.

It is known that not everything which a teacher teaches is learned. “Covering” material does not necessarily mean the students have learned that material. The purpose of this study was to determine if, as a consequence of the HOE verbal problems instruction, a. the students had made significant pre- to post-test gains on the test as whole, b. the gain made by the students for each of the examples was significant, and c. if any problems were particularly difficult for the students and if so, clarify the instructional procedures for those specific problems.

Method:

To ascertain that the students had the algebraic prerequisites which they were supposed to have learned in Level I of the program –and which were needed for the solution of the verbal problems-- the students were provide with two non-verbal questions which involved only the ability to solve algebraic linear equations. They were provided with the following two equations:

1. $3x = x + 4$ $x =$ ____ Check: _____

2. $4x + 3 = 3x + 9$ $x =$ ____ Check: _____

With very few exceptions, the students had both of these examples correct on the pre-test, thereby indicating that they had the algebraic prerequisites with which to solve the verbal problems.

The students were also provided with a six-question verbal problem pre-test designed by the author, to provide the students with a range of distinct verbal problems. Each question was selected randomly from a group of similar questions. In determining the sequence of the questions, those which the author considered simpler algebraically or structurally were placed earlier in the sequence. Thus for example, question #1 could be solved with an equation such $2x + 4 = 18$, whereas question #5 needed an equation such as $4x + 1 = 2x + 11$. In the view of the author, the questions were thus selected in increasing order of difficulty.

Teachers:

After omitting three teachers and their classes (for reasons noted below), 24 teachers and their classes were included in this study. Each of these teachers had received a full day training session on the use of Hands-On Equations to solve verbal problems on May 6th, 2008, some six months prior to teaching the verbal problem lessons to the students and participating in the current study. The reason for this delay (between the time of the training and the implementation of the study) was that May was near the end of the school year. Additionally, time was needed at the beginning of the fall 2008 school year for the teachers to present Level I of Hands-On Equations, so that the students would have the algebraic prerequisites needed to solve the verbal problems.

The large majority of the teachers had more than 10 years of teaching experience. Three teachers had 5 – 10 years of teaching experience and two teachers had 1 - 3 years of teaching experience.

Students:

The students were those in the district who were designated as gifted and who were provided with instruction in pull-out programs. The students were classified by their teachers as suburban students. All of the students in the study had been taught Level I of Hands-On Equations.

Instruction:

The students were given six specific instructional lessons, each one focusing on one of the question types which appeared on the pre-test. There were four instructional examples in each lesson. Two examples were of one type and the other two of a similar, but different type. For example, in Lesson #1 the students were provided with each of these instructional examples, and a second one similar to it:

A. Jane went to the store and spent $16. She bought 3 equally priced cans of food and a $1 container of milk. How much did each can of food cost?

B. Jim had 5 sets of cards (all the sets had the same number of cards) and 2 loose cards. Together, he had 32 cards. How many cards were in each set?

The students were then provided with a worksheet which contained one additional example similar to A, one similar to B, and five other examples which reviewed prior verbal problem lessons or equations. The average time for the instructional component of each of the lessons was 18 minutes and for the worksheet 35 minutes.

Classes of the Study:

Although 27 classes participated in the pre- and post-testing, only 24 of the classes could be used in the study. One class was rejected since it included 4^{th} and 5^{th} grade students; another was rejected because the pre-test score was the highest among all the classes, yet there was hardly any gain over the course of the seven lessons—a situation which did not seem reasonable; a third study was rejected since the teacher indicated that she had not completed Lesson #6.

t-Test Analysis:

For each class a t-test was conducted to determine if the class as a whole had made a significant gain in going from the pre-test to the post-test. Of the 24 classes, 16 had made a significant gain at the .01 level. Six of the classes, each having six or fewer students, did not show a significant t-value, although the average of those classes increased from 38.5% on the pre-test to 68.7% on the post-test. We* suspected that the small class size was hiding the significance. A meta-analysis was conducted on these six classes comprising 26 students. The t-value obtained was significant at the .01 level. Hence of the 24 classes included in the study, only two did not show a significant increase from pre-test to post-test.

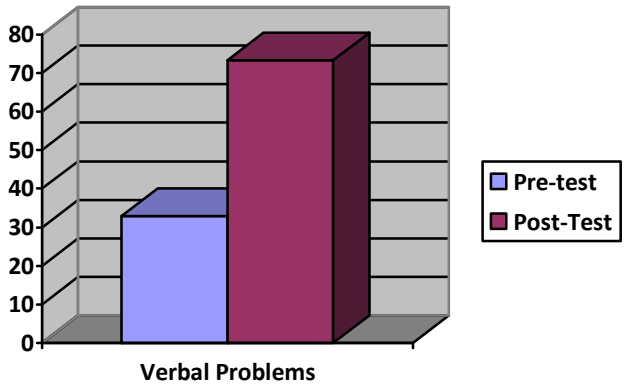
*Larry W. Barber assisted with the statistics.

Hypothesis:

Based on previous studies conducted with Hands-On Equations as applied to the solution of equations, where the results were generally 80% or higher, it was expected that the students would achieve at the 80% level on the test as a whole, as well as on each of the individual items.

Group Result:

For the group of 195 students the pre-test average increased from 1.98 (33%) to 4.40 (73.3%).



The gain obtained by the students, though significant, was slightly less than expected (73.3% vs. 80% or more).

Individual Item Results:

	Pre-test	Post-test	t-value
Questions #1	59%	93%	T=8.25, sig .01
Question #2	55%	95%	T=7.42, sig. 01
Question #3	11%	61%	T=12.98, sig. 01
Question #4	32%	34%	T=.53, not sig.
Question #5	13%	68%	T=14.23, sig .01
Question #6	28%	88%	T=15.65, sig .01

Table 1: Percentage of the 195 students having the correct answer to each question

The same results displayed in a chart are shown below:

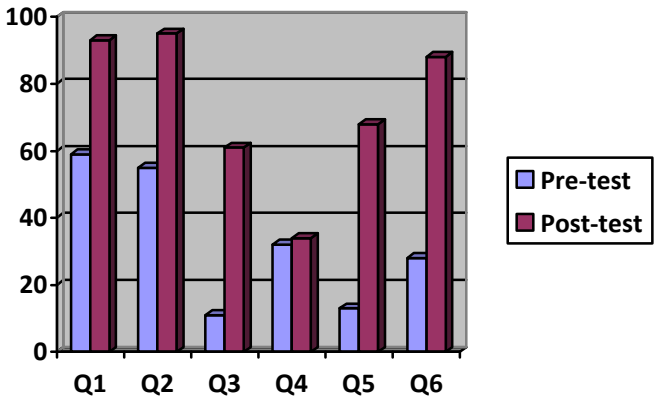


Table 2: Chart showing the percentage of students having the example correct.

On an item analysis for the entire group of 195 students, the gain from pre-test to post-test was significant at the .01 level on all items except for item #4 where almost the same percentage of students had the item correct on the post-test as on the pre-test (34% vs. 32%). Additionally, the results obtained for items #3 and #5 were less than the desired 80%.

In the next sections suggestions will be made for teachers to follow when they next teach this instructions unit so that the at least 80% of the students have each item correct.

Analysis of Items and Recommendations for Future Instruction:

Question #4:

> Josh is 3 years older than three times Darlene's age. If the sum of their ages is 15, how old is Darlene?

The most difficult question for the students was that shown above. 32% of the students had the example correct on the pre-test and only 34% on the post-test. In future instruction it is recommended that the teacher focus on the need to represent in writing the ages of both individuals, and to focus on the aspect of the problem which speaks in terms of the sum of their ages. It is recommended that the students represent the problem in full as shown in the first three steps below. The students need to be sure that the representation for the sum of both ages is placed unto the balance scale.

Josh is 3 years older than three times Darlene's age. If the sum of their ages is 15, how old is Darlene?

Solution:

Let be Darlene's age

Then is Josh's age

Hence, the sum of their ages is

And the equation is

Hence, =3. Ans. Darlene is 3 and Josh is 12. Check: 15=15

Question #3:

When a number is increased by 2, and the sum obtained is doubled, 16 is obtained. What was the number?

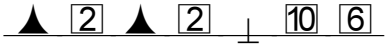
The next most difficult question for the students was question #3 shown above. 11% of the students had the question correct on the pre-test whereas 61% had it correct on the post-test. This gain was statistically significant at the .01 level, but it did not have the 80% success rate that was desired. In future instruction, it is recommended that the teacher focus on what needs to be doubled. It is a sum that needs to be doubled in particular, the sum obtained when the number is increased by 2. In addition, it is recommended that the teacher instruct the students to write down the representation in full as shown in the first three steps below. This will help the students to focus on the component that needs to be doubled.

Solution:

Let be the number

Then, the number increase by 2 is

When this sum is doubled we have

Hence, the equation is 

Hence, =6. Ans. The number is 6. Check: 6 + 2 = 8; when 8 is doubled the result is 15. Hence, 15=15

Question #5:

Four times a number, increased by 4, is the same as twice the number, increased by 12, find the number.

The next most difficult question for the students on the post-test was question #5 shown above. 13% of the students had the example correct on the pre-test and 68% had it correct on the post-test. The gain was statistically significant at the .01 level, although the achievement was not at the desired 80% level. In future instruction it is recommended that the teacher have the students write out the representation in full as shown by the first four steps below. This will help the teacher to discover what aspect of the representation or setup is most difficult for the students.

Four times a number, increased by 4, is the same as twice the number, increased by 12, find the number.

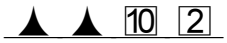
Solution:

Let be the number

Then represents four times the number

And, represents four times the number increased by 4

Twice the number is represented by

And twice the number increased by 12 is 

Summary:

The group of 195 gifted 3rd graders improved in their ability to solve the specific verbal problems presented in this study. The increase from 1.98 (33%) to 4.40 (73.3%) was statistically significant at the .01 level. Furthermore, on each of the examples, except for question #4, there was a significant increase in the percentage of students who solved the problem on the post-test as compared to those who solved them correctly on the pre-test. On the post-test, three of the examples were correctly solved by more than 80% of the students (actually, more than 88% of the students had these three examples correct). Two of the examples were correctly solved by more than 60% of the students, but less than 80%. One example was only solved by 34% of the

students on the post-tests. Instructional suggestions were provided to increase the ability of these students to solve these three examples correct next time around.

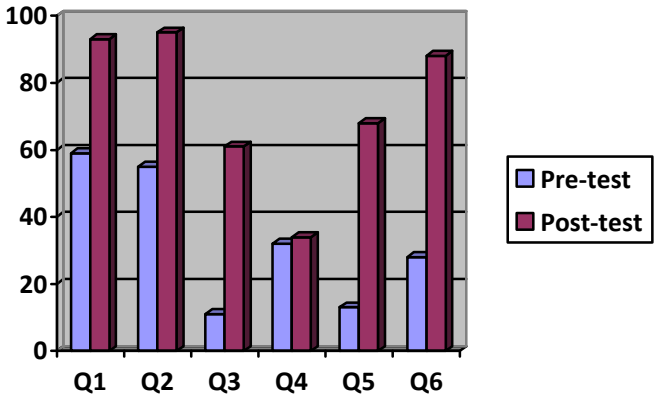


Table 2: Chart showing the percentage of students having the example correct.

The student results, if these should turn out to be representative of other students in grades 4 through high school, would seem indicate a different sequence of question if the intention is to have the questions in increasing order of difficulty, possibly the sequence should be: #1, #2, #6, #5, #3 and #4. This sequence was obtained by taking the average of the pre- and post-test score for each item. We will consider whether future verbal problem studies should list the questions in the above order.

General Conclusion:

This is the first set of verbal problem data to be obtained. The results are encouraging in that it shows a significant gain by these gifted 3rd graders in solving the specific problems under instruction.

Additional studies are needed with other students in grades 3 through high school to see if this method of instruction also leads to successful learning by those students. Future studies should provide at least one or two examples on the pre-test and post-test to test for transfer. Since the methods of instructional present a *general* solution method, transfer of learning would be expected.

Additionally, studies should also be undertaken to compare instruction on these six specific lessons using HOE and a more traditional approach. To be sure that the groups are comparable, the students in both groups should attain similar scores on Level I algebraic equations, which are a prerequisite to the work with verbal problems.

References

1) Barber, L.W. and Borenson, H. Hands-On Equations, Interim Report, Nov 19, 2007: A Comparison of Algebra Achievement by 4^{th}, 6^{th} and 8^{th} graders, Borenson and Associates Inc, 2007.
2) Barclay, J., A study of a Manipulative Approach to Teaching Linear Equations to Sixth Grade Students. (Masters Degree Dissertation), Texas Woman's University, Denton Texas, 1992.
3) Borenson, Henry. The Hands-On Equations Learning System. 2005. Borenson and Associates, Inc. Allentown, PA
4) Borenson, Henry and Barber, L.W. The Effect of Hands-On Equation on the Learning of Algebra by 4^{th} and 5^{th} Grade Students of Broward County, FL. 2008. Borenson and Associates, Inc. Allentown, PA.

Appendix

VERBAL PROBLEMS PRE-TEST A
(PRIOR TO BEGINNING HANDS-ON EQUATIONS®)

Student's Name: _____ Code: _____

Teacher's Name: _____

I am in grade: _____ Today's Date: _____

Instructions to the Student:

You have now had the first seven lessons of Hands-On Equations. Before showing you how to do verbal problems with this program, we would like to see how you do on your own. Feel free to use the game pieces or to use the pictorial notation to solve the equations and the verbal problems shown below. Please write the answer in the space shown. You will have 30 minutes to complete this pre-test. How you do on this test will not affect your grade in any way. Thank you for your participation.

Equations

1. $3x = x + 12$ $x =$ ____ Check: _____
2. $4x + 3 = 3x + 6$ $x =$ ____ Check: _____

1. Charlie went to the store and spent $18. He bought 2 equally priced items and a $4 bag of chips. How much was each item? Answer _____ Check: _____

2. Kathy purchased 5 books for the same amount as Janie, who purchased 2 of the same books and a $9 CD. What was the price of each book?
   Answer _____ Check: _____

3. When a number is increased by 2, and the sum obtained is doubled, 20 is obtained. What was the number? Answer _____ Check: _____

4. Jennifer is 3 years older than twice Julie's age. If the sum of their ages is 9, how old is Julie?
   Answer _____ Check: _____

5. Four times a number, increased by 1, is the same as twice the number, increased by 11, find the number. Answer _____ Check: _____

6. Tom buys 2 packs of football cards to add to the 4 cards a friend gave him. His mother then gives him 3 packs as a present. Now he has as many cards as Billy who owns 2 packs and 13 loose cards. If all the packs have the same number of cards, how many cards are in each pack?
   Answer _____ Check: _____

HANDS-ON EQUATIONS® VERBAL PROBLEMS POST-TEST A

(AFTER COMPLETION OF THE SIX INSTRUCTIONAL UNITS ON VERBAL PROBLEMS)

Student's Name: _____ Code: _____

Teacher's Name: _____

I am in grade: _____ Today's Date: _____

Instructions to the Student:

You have now had six lessons on using Hands-On Equations to solve verbal problems. We would now like to see how you do with the knowledge you gained in these six lessons. Feel free to use the game pieces or to use the pictorial notation to solve the problems below. Please look over the questions and write down the answers to those problems you already know how to do. You will have 30 minutes to complete this post-test. How you do on this test will not affect your grade in any way. Thank you for your participation.

Equations

3. $3x = x + 4$ $x =$ ____ Check: _____
4. $4x + 3 = 3x + 9$ $x =$ ____ Check: _____

1. Jose went to the store and spent $15. He bought 3 equally priced bags of chips and a $3 bar of candy. How much did each bag of chips cost?
   Answer _____ Check: _____

2. Kathy purchased 5 bags of candy for the same amount as Janie, who purchased 3 of the same bags of candy and a $4 bag of potato chips. What was the price of each bag of candies?
   Answer _____ Check: _____

3. When a number is increased by 2, and the sum obtained is doubled, 16 is obtained. What was the number? Answer _____ Check: _____

4. Josh is 3 years older than three times Darlene's age. If the sum of their ages is 15, how old is Darlene? Answer _____ Check: _____

5. Four times a number, increased by 4, is the same as twice the number, increased by 12, find the number. Answer _____ Check: _____

6. Tom buys 2 packs of football cards to add to the 6 cards a friend gave him. His mother then gives him 3 packs as a present. Now he has as many cards as Jane who owns 2 packs and 18 loose cards. If all the packs have the same number of cards, how many cards are in each pack?
   Answer _____ Check: _____