ALGEBRA FOR SIXTH GRADERS: AN INVESTIGATION OF THE PERCEIVED DIFFERENCE IN SUBSEQUENT LEARNING IN ALGEBRA ATTRIBUTED TO THE *HANDS-ON EQUATIONS LEARNING SYSTEM*

By

©2007
Merrie Leah Skaggs
B.A., Baker University, 1967
M.A., University of Kansas, 1979

Submitted to the Department of Curriculum and Teaching and the Faculty of the Graduate School of the University of Kansas in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Dissertation Committee:

____
Chairperson

____

____

____

____

Date defended February 22, 2007

The Dissertation Committee for Merrie Leah Skaggs certifies that this is the approved version of the following dissertation:

ALGEBRA FOR SIXTH GRADERS: AN INVESTIGATION OF THE PERCEIVED DIFFERENCE IN SUBSEQUENT LEARNING IN ALGEBRA ATTRIBUTED TO THE *HANDS-ON EQUATIONS LEARNING SYSTEM*

Committee:

____
Chairperson

____

____

____

____

Date approved ____________________

DISSERTATION ABSTRACT

The purpose of this qualitative study was to examine the perceptions of high school graduates who experienced the mathematics education materials from the *Hands-On Equations Learning System* (Hands-On Equations) when the students were in the sixth grade. The investigation also included the perceptions about mathematics of students who did not experience these manipulative materials.

The participants for this research had attended school and graduated from high school in a small, public school district in eastern Kansas. Of the 19 students who were interviewed, 10 had experienced 21 lessons that involved Hands-On Equations when they were in the sixth grade in January 1997. Ten of the students were male and nine were female. The data consisted of the interviews that were conducted with these students in 2005, solutions to six one-variable linear equations completed by each student, and GPA and ACT information for each student.

Students recalled positive reactions and valued the Hands-On Equations experience. They recommended that other sixth-grade students be taught algebra with these materials. The

reasons for valuing the Hands-On Equations materials included the access to foundational algebraic knowledge that helped students when they got to their first algebra class, alignment with visual or hands-on learning styles, and the promotion of student interest in mathematics.

Several areas of comparison between the two groups of students were analyzed. No obvious difference in present mathematics self-efficacy between the students in the two groups was discerned. Differences were noted when student attitudes were examined. Hands-On Equations students favored mathematics noticeably more than the non-Hands-On Equations students did. The Hands-On Equations group had both a lower mean GPA and lower mean ACT mathematics score, however the students in the Hands-On Equations group solved the six one-variable linear equations with more success (72% accuracy) than did the non-Hands-On Equations group (59% accuracy).

The results from this study confirm the information from other studies (Barclay, 1992; Busta, 1993; Leinenbach & Raymond, 1996) that suggest that Hands-On Equations may help middle level students learn algebra.

TABLE OF CONTENTS

LIST OF TABLES

ACKNOWLEDGMENTS

I want to thank my husband Ron for his infinite understanding and support during all of the years devoted to this doctoral goal. I am also grateful for my children's ongoing encouragement.

Appreciation is extended to the chairperson and past and current members of my committee, Dr. Susan Gay, Dr. Robert Hohn, Dr. Nona Tollefson, Dr. Fred Van Vleck, Dr. Jerry Bailey, Dr. Betty Bullock, Dr. Phil McKnight, and Dr. Suzanne Rice for their guidance, expertise, and help at various stages of my graduate program.

CHAPTER 1

THE RESEARCH PROBLEM

Introduction

Algebra in the elementary school builds a foundation of understanding for later "more-sophisticated work in algebra in the middle grades and high school" (National Council of Teachers of Mathematics [NCTM], 2000, p. 37). Traditionally, students first enroll in algebra courses in middle school or high school and bring little or no prior knowledge of the topic. This abrupt transition is problematic for many students (Greenes & Findell, 1999).

Algebra in the elementary school is a relatively new reality. National mathematics standards now require the inclusion of algebra in elementary classrooms. NCTM (2000) revised the national mathematics standards for school mathematics in the document *Principles and Standards for School Mathematics.* These standards describe an emphasis on algebra from prekindergarten through grade 12. Earlier national recommendations for an algebra strand in elementary school also exist (Educational Testing Service and the College Board, 1990; NCTM, 1989, 1992).

Numerous states list algebra in the standards for elementary grades. Frequently, states have followed the lead of NCTM and have written state standards that align with the national standards. For example, the Alaska standards (Alaska Department of Education & Early Development, n.d.) include the heading “Functions and Relationships” in the standards for students aged 5-7. Students aged 8-10 study functions as well as the patterns that were included for 5-7-year-olds. The Arizona standards (Arizona Department of Education, 2003) are organized by strands; Strand 3 is titled “Patterns, Algebra, and Functions” and students in the lower grades focus on patterns, functions with T-charts, finding the missing element in algebraic representations, and the change in a variable over time. In the state of Kansas, the mandate that teachers must teach and students must learn algebra is especially strong since elementary students are tested over algebraic understandings on the Kansas Mathematics Assessments (Kansas State Department of Education, 2001). These assessments are based on the *Kansas Curricular Standards for Mathematics* (Kansas State Board of Education, 2004).

Greenes and Findell (1999) asserted that a mechanism is needed to give students early experiences with algebra. These experiences should continue throughout all stages of their mathematical development. Greenes and Findell stated that a “. . .lack of experience has occurred despite the fact that several professional groups have recommended that algebra be a curricular strand in kindergarten through grade 8 mathematics programs” (p. 127).

Hands-On Equations Learning System

The *Hands-On Equations Learning System* (hereinafter referred to as Hands-On Equations) makes algebraic concepts concrete and thus attainable for all in the third grade on up (Borenson, 1994). Hands-On Equations is a set of instructional materials that includes student and teacher manipulative materials, three levels of teacher manuals that explain the 26 lessons, and worksheets for each lesson. The system "provides concrete and invaluable experience in using the basic ideas associated with algebraic linear equations" (Borenson, 1994, p. 23). Borenson (1994), the creator of these materials, claimed that the system imparts important mathematical content, promotes mathematical interest, and heightens student self-esteem.

Statement of the Problem

The purpose of this study was to examine the perceptions of high school graduates who experienced the mathematical materials from Hands-On Equations when the students were in the sixth grade. The investigation also included the perceptions of students who did not experience Hands-On Equations during their sixth-grade year. Four research questions were addressed.

1. For the students who experienced Hands-On Equations, what is the perceived value of these materials?
2. Did the Hands-On Equations lessons create student perceived differences in subsequent learning in algebra classes for students taught with Hands-On Equations?
3. Is there a difference in present mathematics self-efficacy between students taught with Hands-On Equations and those who did not experience these teaching materials?
4. Are there other differences related to (a) student attitudes toward mathematics, (b) student achievement in mathematics, and (c) student ability to solve simple linear equations between students taught with Hands-On Equations and students who were not?

Two groups of students who graduated from Baldwin High School in 2003 were interviewed. Each member of both groups of students was enrolled in the Baldwin USD 348 school district from at least the 6th through the 12th grade. The first group consisted of 10 students who were in a sixth-grade classroom in which 21 lessons pertaining to Hands-On Equations were taught in January 1997. The second group was composed of 10 students who were in sixth-grade classrooms where Hands-On Equations was not used. One of the non-Hands-On Equations students was later omitted from the study when it was realized that she did not meet the criteria of attending Baldwin schools from the 6th grade through the 12th grade. As a result, 19 interviews were included in the data.

Rationale for the Study

Kansas and national standards now require that algebraic concepts be taught to elementary school students (Kansas State Board of Education, 2004; NCTM, 2000). The National Council of Teachers of Mathematics (2000) stated, "All students should learn algebra" (p.37). All students are defined as students in prekindergarten through 12th grade (NCTM, 2000).

One reason for the inclusion of algebra in these standards could be the fact reported by the National Research Council

(1989) that over 75% of all jobs require the use of algebra in either a qualifying examination or in basic proficiency in order to perform the job. Lawson (1990) and Carifio and Nasser (1994) explained that algebra is the entry level skill in most technical jobs, industry, sciences, and business. The National Research Council (1989) also described a 50% attrition rate per year among mathematics students from the ninth grade through the Ph.D. level; this translates into half of the mathematics students per year dropping out of mathematics classes.

Algebra as a mathematical topic in elementary school looks different from the more formal algebra of middle school or high school but involves the same basic understandings (NCTM, 2000). *Principles and Standards for School Mathematics* (NCTM, 2000) stated common goals in algebra for all students from prekindergarten through grade 12. They should know and be able to understand patterns, relations, and functions; represent and analyze mathematical situations using algebraic symbols; use mathematical models; and analyze change. Additional sets of expectations that are specific to each grade band (prekindergarten-2, 3-5, 6-8, 9-12) delineated the differences among grade levels.

Nationally, specific components of algebra are assigned to various grade levels. Students in grades prekindergarten through grade 2 should recognize patterns, sort and classify objects, and use pictorial representations as a precursor to conventional symbolic notations (NCTM, 2000). Students in grades 3 through 5 should make generalizations about geometric and numeric patterns, represent a variable as an unknown with a letter or symbol, and look at constant or varying rates of change.

At the state level, expectations also vary depending on the grade level. For example, the *Kansas Curricular Standards for Mathematics* (Kansas State Board of Education, 2004) expects kindergarten children to look at patterns and to find unknown sums and represent these sums with concrete objects and pictures. First graders should state functional relationships in either vertical or horizontal function tables for whole numbers 0 through 50. By fifth grade, students should be able to solve one-step linear equations such as 3y=12. Students at this grade level continue to express functions in tabular form but they deal with numerical patterns in numbers as large as 5000.

While the degree of difficulty increases through the grades, algebra as a strand is included in the curriculum beginning with prekindergarten and builds a foundation for the formal study of algebra in middle school or high school in both state and national standards (Kansas State Board of Education, 2004; NCTM, 2000). Traditionally, students in a middle school or high school Algebra I course have not previously encountered the basic ideas of algebra before that first formal course; they falter because they are expected to assimilate algebraic ideas and skills in a very short time (Greenes & Findell, 1999; Von Rotz & Burns, 2002).

Von Rotz and Burns (2002) gave examples where students use abstract symbols without understanding. For example, young children frequently misinterpret the meaning of the equal sign. They often assume that this symbol acts as a sign that the answer comes next. Thus children will write 7 for the box in a problem such as $2 + 5 = \square + 4$. The basic concept of equality is misunderstood when children will accept $3 + 4 = 7$ as correct but reject $7 = 3 + 4$.

Hands-On Equations is a set of concrete materials specifically designed to meet the algebraic needs of teachers and students in the elementary grades. Fennema and Franke (1992) asserted that concrete representations, as well as real-world situations and pictorial representations, help students learn abstract concepts in mathematics. Phi Delta Kappa supported

Hands-On Equations for years; for example, Phi Delta Kappa (2000) promoted Hands-On Equations by advertising and sponsoring 102 workshops during the spring of 2000. These workshops were titled, “Making Algebra Child’s Play! The One-Day Hands-On Equations Workshop.”

Even though Hands-On Equations has been in existence since the early 1990s, a survey of the literature found minimal research on Hands-On Equations. This absence of research provided a rationale for the current study. The present investigation could fill an obvious void in the literature, and could provide information about the potential contribution of Hands-On Equations as a tool to introduce beginning algebraic concepts and to enhance student success in and attitudes toward algebra.

Definition of Terms

For the purpose of this research study, the following definitions will be used.

Attitude toward mathematics refers to characteristics including “a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless” (Neale, 1969, p. 632).

Hands-On Equations refers to the manipulative materials, *Hands-On Equations Learning System*, developed and patented by Henry Borenson (1994).

Middle School, Middle Level, and Junior High School are terms that are used interchangeably to denote grades 6 through 8.

Self-efficacy is defined as the self assessment of one's capability to succeed to a certain level in specific subject areas (Bandura, 1986).

Assumptions

It was assumed that students who attended school in Baldwin from at least 6^{th} grade through 12^{th} grade had similar experiences except for academic tracking that might have occurred. From 1997 until these students graduated from high school in 2003, the various routes through a mathematical curriculum were not specifically prescribed. Students were required to take two year-long mathematics courses of their choice. Students who were academically stronger typically took Advanced Mathematics I their senior year; academically weaker students who took Applied Math I and II could graduate from high school without a formal algebra course (Baldwin High School, 1998). It was assumed that the students with more opportunities to learn algebra would be more successful in solving the six equations that were a component of the interviews.

The researcher assumed that the students willingly participated in this study. Another assumption was that the students felt comfortable enough to answer questions honestly and thoroughly. The researcher who conducted most of the interviews knows many of the students who were interviewed and assumed that these relationships would contribute to student comfort levels. A second interviewer conducted three of the interviews to avoid potential bias. It was assumed that there was no effect on the data as a result of using two interviewers. An additional assumption was that the students' memories were at least somewhat accurate. It was assumed that the students with higher academic achievement records would remember more.

Limitations

This investigation involved students in one school district. The study was conducted 8 years after the students first experienced the Hands-On Equations lessons. Their subsequent exposure to mathematics classes may or may not be typical of other students who matriculate through junior and senior high schools elsewhere. Consequently, the research results may not be generalizable to students in other school districts.

The researcher and the teacher of the 1997 Hands-On Equations lessons are the same person. This fact introduced a potential bias so that students may have told the researcher what they thought the researcher wanted to hear during the interviews.

The Hands-On Equations lessons conducted in 1997 were taught by a teacher licensed to teach mathematics at the elementary school, junior high school, and high school levels and who had attended a training day on Hands-On Equations. Other teachers of these materials may or may not have an interest in or understanding of the mathematical concepts inherent in these materials. As a result, others attempting to replicate these research findings may not be able to locate teachers with the necessary mathematical background to adequately teach Hands-On Equations.

Memory is another potential limitation. Since this study was retrospective and asked students to recall reactions from prior years in their academic careers, faulty memory may have been a factor in the study.

Overview

This study is divided into five chapters with the first chapter introducing the problem. A review of the related literature is found in Chapter 2. Pertinent topics included there are the abstract nature of algebra, student achievement in algebra, student difficulty with learning algebra, learning theories and theorists, the value of manipulatives, Hands-On Equations, memory and long-term remembering, self-efficacy and mathematics, and attitude toward mathematics. Chapter 3 describes the qualitative research procedures that were conducted during the study. Background information on an informal study conducted with the sixth grade during 1997 is also included. The chapter concludes with the results of a pilot study. Chapter 4 includes the results of the analysis of the data accumulated during the study. Chapter 5 presents a summary, conclusions, and recommendations for practitioners and researchers.

CHAPTER 2

REVIEW OF THE LITERATURE

Introduction

This chapter contains a review of the literature pertaining to topics that are relevant to this study. The review starts with a discussion of the abstract nature of algebra. The review then looks at student achievement in algebra and student difficulty with learning algebra. The review of the research next considers learning theories and the roles of concrete and abstract learning including the relationship between concrete learning and the use of manipulatives such as Hands-On Equations. The limited research available on Hands-On Equations is addressed next. Long-term remembering is also a factor and is considered. Finally, the review looks at self-efficacy and attitude toward mathematics.

The Abstract Nature of Algebra

The abstract, symbolic nature of algebra is apparent when the various definitions of algebra are reviewed. The *American*

Heritage College Dictionary (Costello, 1993) defined algebra as "a generalization of arithmetic in which symbols, usu. [usually] letters, represent numbers or members of a set of numbers to which the same operations apply" (p. 33). Carlyle and Moses (as cited in Tierney & Nemirovsky, 1997, p. 336) said, "Algebraic thinking encourages examination or conceptualization of number relationships in general. It is a way of going from specific thinking to general cases." MacGregor and Stacey (1999) stated, "Algebra is that part of mathematical language that has been designed to express general relationships among numbers" (p. 85).

NCTM's (2000) Standard 2: Algebra is delineated by four components describing what students should know and be able to do in regards to algebra. Teachers should teach so that students will

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts. (p. 37)

Esty (1999) offered distinctions between arithmetic and algebra. With arithmetic, one can go *directly* to the solution of a problem. With algebra, a problem is represented by symbols that one must manipulate, thus *indirectly* finding the solution. Algebraic notation is needed when "the problem is indirect" (p. 141). An example he offered exemplified his point. Esty's accompanying diagram showed a marked side which was one of three sides and was perpendicular to the barn wall.

> Problem 1: A farmer builds a rectangular dog kennel along the side of her barn, so she uses only three sides worth of fencing and gates. If she uses 40 feet of fencing and gates, with the marked side projecting 7 feet from the barn, what is the area of the kennel?
>
> Problem 2: A farmer builds a rectangular dog kennel along the side of her barn, so she uses only three sides worth of fencing and gates. If she uses 40 feet of fencing and gates and the area is 170 square feet, how long is the marked side? (p. 141)

In the first problem, one can simply do the calculations, $40-7-7=26$, in order to find the length of the dog kennel. The length of 26 feet times the width of 7 feet equals an area of 182 square feet; algebra is not required to solve this area problem. In the second problem, there is nothing to calculate at first. Algebra is needed to create a formula, $170 = x(40-2x)$, that can then be solved to determine the length of the marked side. Esty emphasized that "algebraic thinking is characterized by its focus on operations and order, as opposed to an emphasis on numbers" (p.144). He asserted that the realities of algebra are operations, order, and relations.

In going from the specific to the general, formulas are often derived. Herbert and Brown (1997) explained how children as young as sixth grade can recognize the power of an algebraic formula. They shared one student's comments after working on the "Crossing the River" problem. Students were to find out how many one-way trips across a river it would take to move eight adults and two children across a river in a boat that could hold either one adult, or one child, or two children, if each of them could row the boat. Students were urged to follow three phases: pattern seeking, pattern recognition, and generalization. After generalizing, one student said, "I think we try to find formulas so it will be easier to get the problems done. Formulas make problems easy to solve. It's very helpful" (p.343).

Saul (2001) quoted Sir Isaac Newton as an authority on the definition of algebra. Newton explained that algebra is a generalization of arithmetic, or the *arithmeticae universalis;* ". . .whereas arithmetic treats questions in a definite, particular way, algebra does so in an indefinite universal manner. . ." (p. 38).

Student Achievement in Algebra

NAEP Results

Kenney and Silver (1997), as co-director and director of the National Assessment of Educational Progress (NAEP) Interpretive Reports Project, probed the foundations of algebra by examining fourth-grade students' responses on the 1992 NAEP. Approximately 10% of the fourth-grade items assessed algebraic thinking. "Algebraic thinking" involved patterns of figures, symbols, or numbers. Both multiple-choice and constructed-response items were included. Kenney and Silver's qualitative examination of 250 student responses revealed that the NAEP assessed important, informal algebraic concepts related to patterns and relationships. The NAEP data also revealed these students' ability to work "with patterns of numbers, where the relationship between the numbers was explicitly presented or implicitly defined" (p. 274).

Specifically, Kenney and Silver (1997) found that fourth graders could reason with simple patterns but had more trouble with complex patterns and explaining their mathematical reasoning about patterns. For example, a simple problem involving repeated figures that asked the student to determine the next figure was correctly answered by almost all fourth-grade students. In contrast, student responses to the "Extend Pattern" question, which required students to determine if 375 would appear in the sequence generated by multiplying by 2s, showed that while over 75% of the students chose the correct answer, only about 25% wrote a mathematically valid reason to explain the answer. Student problems continued when they were asked to determine a pattern of "decreasing increases" (p. 270). Explicitly stated patterns or patterns that increased by a constant appeared to be easier for fourth-grade students than those patterns with non-constant differences. Another example offered by Kenney and Silver revealed that students confused the role of the input and output columns in a table. Also, while students could sometimes decipher the pattern, they could not consistently apply it to a new situation. Kenney and Silver suggested that more explicit attention to patterns is needed in elementary school classroom teaching. They claimed that much more practice with these types of patterns could enhance student achievement on future NAEP tests.

In looking more closely at the NAEP algebra and functions question data for fourth-grade students from *The Nation's Report Card* (National Center for Education Statistics, n.d.), it is apparent that a majority of the students in years 1990, 1992, 1996, or 2003 correctly answered five out of seven of the procedural knowledge questions for which data was reported. In contrast, a majority of these students correctly answered only two out of seven of the conceptual-understanding questions and only four out of eight of the problem-solving questions. This set of data confirms the pattern for American students of performing well on computational tasks but not achieving as well on questions dealing with deeper mathematical understanding (Kilpatrick, Swafford, & Findell, 2001).

Kansas State Assessment Results

The Kansas State Department of Education (Kansas State Department of Education, n.d.) organizes its mathematics standards hierarchically by benchmarks and indicators that are denoted as either knowledge or application indicators. The numbering system on the retrieved reports indicates these categories. For example, K2.4.1 is the label for Knowledge

Indicator l in Standard 2 under Benchmark 4. An application indicator example is A2.3.1 which denotes Application Indicator 1 in Standard 2 under Benchmark 3. (While the retrieved reports followed this labeling system, the current standards use a pattern that inserts the "K" or "A" next to the indicator it describes, for example 2.4.K1 or 2.3.A1.) The knowledge and application indicators for Standard 2 Algebra specify algebraic achievement objectives.

By comparing the scores (Center for Educational Testing and Evaluation, n.d.a; Center for Educational Testing and Evaluation, n.d.b) of Kansas fourth-grade students who took the state assessment in mathematics in spring 2000 and in spring 2005, one can determine if Kansas algebra scores have improved or not during this 5-year period. Utilizing the six algebra indicators, K2.1.1, K2.3.3, K2.4.1, A2.1.2, A2.2.2, and A2.3.1, the state means for 2000 were 74.5, 48.9, 53.5, 57.7, 52.3, 51.2, respectively. The matching state algebra means for 2005 were 77.6, 72.7, 71.4, 70.2, 53.9, and 52.8. These data indicate an improvement in algebraic achievement for Kansas fourth graders from 2000 to 2005.

TIMSS Results

The *Third International Mathematics and Science Study* (TIMSS) was "the first large-scale international study integrating information on curriculum and teaching ever attempted" (U.S. National Research Center, 1996). This study was conducted in 1994-1995 at five grade levels in more than 40 countries (International Study Center, 2005). The U.S. National Research Center (1996) explained that this study focused on children aged "9, 13, and those in the last year of high school. In the U.S., these are 4^{th}-, 8^{th}- and 12^{th}-graders." TIMSS was based on the premise that curriculum and teaching methods are factors in what students learn (U.S. National Research Center, 1996).

The TIMSS results revealed two false assumptions: the content of mathematics does not vary across countries and the content follows a fixed sequence (U.S. National Research Center, 1996). Those assumptions were disproved when it was found that mathematics subjects in all countries are not universal and that the various curriculums did not follow an expected sequence. Also, the degree of attention and the type of focus varied as did the types of skills that students were expected to demonstrate.

One of the key findings about curriculum was that United States standards are “unfocused and aimed at the lowest common denominator. In other words, they are a mile wide and an inch deep” (U.S. National Research Center, 1996). Another finding was the fragmented curriculum that is the result of the decentralized approach to education in the United States. Other countries achieved better because of their coherent goals and universal teaching practices. United States students in the eighth grade study arithmetic, fractions, and a small amount of algebra in contrast to both Japan and Germany whose students receive thorough exposure to both algebra and geometry. These findings help to explain the achievement results that TIMSS reported.

The TIMSS fourth- and eighth-grade tests dealt with algebra; the fourth-grade test had a subtest related to algebra called “patterns, relations and functions” and the eighth-grade test was a “full-scale algebra subtest” (U.S. National Research Center, 1997, figure 6). The average percent correct was reported for each of the TIMSS countries (U.S. National Research Center, 1997). The United States was in the middle tier for both fourth- and eighth-grade results which meant that those scores were not significantly different from the mean. Fourth graders scored 66% correct; a 73% was significantly above the mean and scores of 60% and below were significantly below the mean. Eighth graders scored 51% correct; a 59% was significantly above the mean and scores of 46% and below were significantly below the mean.

Student Difficulty with Learning Algebra

Two areas that impact student learning are the content and the students (Kieran, 1992; Kilpatrick, Swafford, & Findell, 2001). Kieran applied these categories specifically to algebra whereas Kilpatrick et al. referred to mathematics in general. "The effectiveness of mathematics teaching and learning is a function of teachers' knowledge and use of mathematical content, of teachers' attention to and work with students, and of students' engagement in and use of mathematical tasks" (Kilpatrick et al., 2001, pp.8-9). The editors of *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick et al., 2001), included these same two areas—the content and the students—in the elements that impact student mathematical proficiency.

Content

Various authors make the case that the content of algebra is inherently difficult. Hatfield, Edwards, Bitter, and Morrow (2005) stated, "What makes algebra seem difficult to some people is that algebra is really two things at once" (p. 410).

They claimed it is both a language and an abstract system with specific rules. Usiskin (1996) explained that mathematics is a symbolic language with the symbols of mathematics forming the written language of mathematics. Esty (1999) wrote an entire textbook and course titled *The Language of Mathematics*; he capitalized Mathematics to emphasize its similarity to other foreign languages.

Esty (1999) explained that Mathematics, like other languages, "has its own vocabulary, grammar (classes of words and rules of arranging them), syntax (rules of word order), synonyms, negations, conventions, abbreviations, sentence structure, and paragraph structure" (p.1). He suggested that students have problems trying to learn this language without having it taught explicitly. Kieran (1989) observed that "this particular aspect of algebra appears to be one that never really does get sorted out by most students throughout their entire high school algebra career" (p.39). Esty (1992) explicitly teaches the language of Mathematics and claimed that the language aspects are essential to the understanding of algebra as well as other areas of mathematics. One of the confusing aspects of this language is the fact that the same symbols are employed to mean different things (Esty, 1992). An example and explanation that Esty offered is:

> Mathematics uses the same symbols ("x" and "=") to discuss numbers that it uses to discuss arithmetic operations.
>
> - "$3(x + 4) = 30$" gives information about x, an unknown number.
> - "$3(x +4) = 3x + 12$" uses the same symbols but gives no information about x; its content is about an entirely different kind of mathematical object, a sequence of operations (function). It says "Add 4 and then multiply by 3" is equivalent to "Multiply by 3 and then add 12."
> - "$3(a + 4) = 3a + 12$" uses a different letter but says the same thing!
>
> This helps explain why Mathematics is difficult. (Esty, 2000, p.1)

Usiskin (1992) stated that algebra is a language that has symbols that stand for both elements and operations on those elements. Usiskin added that as a language, three facts about language apply to algebra:

- It is best learned in context.
- Almost any human being can learn it.

- It is more easily learned when one is younger. (p. 27)

Kieran (1992) extensively studied the difficulties of going from arithmetic to algebra. Her explanation of the dichotomy of algebra involves two terms, *procedural* and *structural*. The first of Esty's examples, $3(x + 4) = 30$, could be labeled procedural by Kieran's definition because it involves "arithmetic operations carried out on numbers to yield numbers" (p. 392). The expression $3(x + 4)$ is indeed a number that is equivalent to 30 in Esty's first example. Kieran explained that the term structural "refers to a different set of operations that are carried out, not on numbers, but on algebraic expressions" (p. 392). The latter two of Esty's examples, $3(x + 4) = 3x + 12$ and $3(a + 4) = 3a + 12$, could be labeled structural because they refer to sets of operations carried out on algebraic expressions.

Kieran's (1992) analysis of the research related to the learning of algebra supported her overall conclusion that the "majority of students do not acquire any real sense of the structural aspects of algebra" (p. 412). She added that as of her 1992 writing, few, if any, textbooks at that time explicitly dealt with helping students make the transition from the procedural to the structural aspects of algebra.

Bruner (1960) wrote about the importance of learning the structure of a subject. “Grasping the structure of a subject is understanding it in a way that permits many other things to be related to it meaningfully” (p. 7). He used an algebra example to make his point. Bruner said that once a student grasps the fundamental ideas of equation solving, he or she can recognize that new equations are “not new at all, but only variants on a familiar theme” (p. 8). Hohn (1995) summarized Wertheimer’s position that students must be assisted to see the structure of problems in order to apply their learning to new situations.

Rittle-Johnson and Alibali (1999) researched the relationship between conceptual and procedural knowledge. Their findings suggested that by fourth grade, most elementary students understand what it means for two quantities to be equal. The issue is the students’ lack of comprehension of the meaning of the equal sign or the structure of equations. Children do not extract the full meaning of the equal sign even after multiple uses. “Indeed, without a prior understanding of equivalence, algebraic equation-solving procedures may not make sense” (p.187) and may lead to difficulties in algebraic equation-solving. “Their incomplete understanding of the meaning and role of the equal sign may be one source of these difficulties" (p.187).

A question that the research of Rittle-Johnson and Alibali (1999) directly addressed was which type of knowledge, procedural or conceptual, should be taught first. They examined fourth and fifth graders' performance with problems of the form $a + b + c = \_ + c$. They found that conceptually oriented instruction produced gains in both conceptual and procedural knowledge; procedurally oriented instruction also produced gains in both types of knowledge with smaller gains in conceptual knowledge. The results indicated that conceptual instruction should precede procedural instruction.

Students

In *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick et al., 2001), the editors shared the Mathematics Learning Study Committee's findings. They confirmed that many students have difficulty in making the transition from "school arithmetic to school algebra—with its symbolism, equation solving, and emphasis on relationships among quantities" (p. 8). Rubenstein and Thompson (2001) summed up student problems based on the inherent nature of algebra. "The symbolic language of mathematics often challenges our students.

We sometimes forget that the words, phrases, and symbols that are meaningful to us are unfamiliar to students" (p. 265). They asserted that students who do not master the standard symbolism of mathematics will be hindered at some point in their mathematical careers (Rubenstein & Thompson, 200l).

Research shows that many students do not understand the foundational idea of equality (Kieran, 1992; Siegler, 1998). Faulkner, Levi, and Carpenter (1999) described an activity where teachers of first- through sixth-grade students asked their students to complete the equation: 8 + 4 = _ + 5. Fewer than 10% of their students answered correctly. A majority of the incorrect answers were 12 with some students answering with 17. Kilpatrick et al. (2001) said that these students exemplify many if not most elementary students who have decided that the equals sign is the signal to supply an answer, that is, to calculate with the numbers that precede the equals sign. "Children can develop this impression because that is how the notation is often described in the elementary school curriculum and most of their practice exercises fit that pattern" (Kilpatrick et al., p. 379). Thus, students do not understand the symbol as an indicator of equality (Behr, Erlwanger, & Nichols, 1976; Erlwanger & Berlanger, 1983; Kieran, 1981; Saenz-Ludlow & Walgamuth, 1998). In addition, teachers often fail to realize the degree of their students' misunderstanding of the equality concept (Faulkner et al., 1999).

Carraher, Schliemann, Brizuela, and Earnest (2006) asserted that empirical studies delineating success with algebra for young students negate the argument that young children are developmentally incapable of comprehending algebraic ideas. A master's thesis written in Spanish by Lins Lessa in 1995 is one study that Carraher et al. cited. Lins Lessa found that 11- to 12-year-olds could use a balance scale to solve problems such as $x + y + 70 = 2x + y + 20$. After citing numerous studies including their own research, Carraher et al. concluded that students' difficulties with algebra may more closely relate to the "limited ways that they were taught about arithmetic and elementary mathematics" (p. 92) rather than a developmental inability. Brizuela and Schliemann (2003) described classroom data that showed that, "if children are given the opportunity to discuss algebraic relations and to develop algebra notations, even fourth graders will be able to solve algebra equations" (p. 1).

A lack of opportunity to learn algebra may be a real issue for students. The NAEP data bank revealed how often fourth-grade teachers addressed algebra and functions (National Center for Education Statistics, n.d.). The self-reported results indicated that 8% spent “a lot” of time, 31% spent “some” time, 41% spent “a little” time, and 20% answered “none” in response to this question. This data showed that 61% of the fourth-grade teachers spent little or no time on algebra and functions.

Another aspect of the lack of opportunity for students involves exposure to appropriate teaching. The TIMSS study provided insights on eighth-grade mathematics teaching in the United States with its 1995 video study component (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). While the reform movement in mathematics education was predominant during the 1990s with mathematics educators, what the TIMSS video study revealed was that this reform movement had not materialized in eighth-grade classrooms across America; teachers were still utilizing traditional teaching methods that did not emphasize high-level thinking and understanding. Twenty years after the studies done in the 1970s, teaching had changed little in American classrooms.

Commenting about the TIMSS results, Lane (1996) stated that United States mathematics teachers rarely have time to teach any subject in depth because they are expected to teach such a wide range of subjects. Also, United States teachers spend more time in the classroom which results in less time to plan lessons. Teacher and public expectations were also noted; other nations contrast with the United States because the United States expects all students to achieve a quality mathematics education rather than an elite few.

Textbooks and the curriculum they relay make a difference in student learning (Schmidt, 1996). The Third International Mathematics and Science Study (TIMSS) reinforced this point.

> Our [United States] unfocused curricula and textbooks fail to define clearly what is intended to be taught. They influence teachers to implement fragmented learning goals in their classrooms. They emphasize familiarity with many topics rather than concentrated attention to a few. Our curricula, textbooks, and teaching are all a "mile wide and an inch deep." (¶ 5)

Superficial understanding by students may lead to incorrect extensions of correct rules as students try to make sense of algebra (Siegler, 2003). An example is the incorrect application of the distributive principle. Some students inaccurately concluded that the distributive property that allows $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ would lead to $a + (b \cdot c) = (a + b) \cdot (a + c)$ (Siegler, 1998). Siegler (1998) noted that a lack of understanding of algebra leads capable and competent students to view algebra as "exercises in symbol manipulation, without any connection to real-world contexts" (p. 296). Mayer (1999) warned that procedural knowledge that is isolated from conceptual knowledge results in mathematics that becomes a set of meaningless procedures for students.

Learning Theories and Theorists

Educational psychology is a science that is a branch of psychology that investigates both the instructor's manipulation of the environment and changes in the learner as a result of that environment (Mayer, 1999). "As a discipline, educational psychology is poised between teaching and learning (i.e., between the instructional manipulations provided by the teacher and the changes in knowledge and behavior created in the learner)" (p. 5). Mayer explained that the definition of educational psychology raises the question of what is learned—behavioral change or a cognitive change—and leads to the "classic tension between behaviorist and cognitive approaches to learning" (p. 5).

Behaviorism versus Cognitivism

Behaviorism involves determining the relationship between two factors, instructional manipulations and outcome performance; both are externally observable events (Mayer,1999). The phrase "instructional manipulations" refers to the stimulus and "outcome performance" refers to the response to that stimulus. With behaviorism, "the goal of educational psychology is to determine how instructional manipulations affect changes in behavior" (p. 7).

A cognitive approach involves the relationship between external factors (the stimulus and response) and internal factors such as learning processes and existing learner characteristics (Mayer, 1999). Thus a main interest of cognitive psychologists is to discover the internal cognitive processes and states that allow understanding of the relationship between instructional manipulations and outcome performances.

Key Ideas About Behaviorism

Behaviorism had its roots in the early 1900s with the work of John B.Watson (1925). Watson was interested in having psychology focus more on observable behavior to the exclusion of consciousness. He was influenced by Ivan Pavlov, the Russian physiologist who studied the responses of dogs to the ringing of bells and other actions in association with food. Watson believed that human learning could be totally controlled through a conditioning process that was similar to that used by

Pavlov. Hohn (1995) gave an education-related application of this conditioning process: The use of flash cards when teaching children math facts or vocabulary words is an example.

Hohn (1995) credited Thorndike with the first theory of learning because his theory was based on organized research. Thorndike conducted numerous systematic experiments with animals and their learning of relatively simple tasks. While he recognized that human learning is more complex than animal learning, Thorndike believed that humans learn in much the same way as animals do. His work gave behaviorism its scientific basis. Hohn indicated that Thorndike's "lasting legacy" (p. 23) is the practice of using research to guide educational practice.

Key Ideas About Constructivism

Lemlech (2002) explained, "Constructivist learning theory is an approach to teaching and learning which acknowledges that information can be transmitted but understanding must be constructed"(p. 130). While von Glasersfeld (1995) is recognized as a leader in this movement, he credited Piaget and others who came much earlier with the genesis of these ideas. Von Glasersfeld clearly stated that constructivist thought is in opposition to the behaviorism that was popular in the twentieth century. Von Glasersfeld defined constructivism in his opening chapter.

> What is radical constructivism? It is an unconventional approach to the problems of knowledge and knowing. It starts from the assumption that knowledge, no matter how it be defined, is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. (p. 1)

Eisner (1999) recognized the emergence of constructivism in the education community's view of human nature. "We have come to realize that meaning matters and that it is not something that can be imparted from teacher to student. Meanings are not given, they are made" (p. 658). Flavell (1992) admitted that developmentalists are in agreement that children are constructive thinkers and learners. Battista (1999) claimed that constructivism had become the "dominant theoretical position among mathematics education researchers" (p. 432). He explained that the research showed that constructivism is based on tested theory.

Von Glasersfeld (1995) shared his radical constructivism at the Eleventh International Conference on the Psychology of

Mathematics Education in Montreal in 1983 and received the attention of the intellectual community. The two basic principles of his model are:

- Knowledge is not passively received but built up by the cognizing subject;
- The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality. (p. 18)

Von Glasersfeld explained that concepts must be built up by the individual; the concepts are not inherent in things. Physical materials are useful but only because they provide opportunities to reflect and abstract. "Reflective abstraction is not a matter of looking closely but of operating mentally in a way that happens to be compatible with the perceptual material at hand" (p. 184). He used Cuisenaire rods as an example, explaining that the rods are not evident manifestations of concepts but are the objects that may direct the attention of the learner and invite construction of mathematical concepts. While the materials "cannot determine the students' conceptual constructing, it can set up constraints that orient them in a particular direction" (p. 184). He emphasized that students often make abstractions that differ from the ones that appear obvious to the teacher. This is because the student is the one who creates, or constructs, the knowledge. Thus the materials may be conducive to the desired understanding but they are "merely occasions, not causes" (p.185).

Related Components from Piaget and the Concept of Developmental Stages

Constructivist thought is influenced by the work of Piaget. Piaget's developmental learning stages have been referenced repeatedly, for example, by Flavell (1992) and Siegler (1998). Hohn (1995) listed these stages as sensorimotor (birth to 2 years), preoperational (2-7 years), concrete operational (7-11 or 12 years), and formal operational (12-14 years to adulthood). Hohn explained that in the concrete operational stage, "the child can manipulate concrete events and can solve problems using them. Game rules are followed" (p. 68). Hohn included 12-year-olds in his description of the formal operational stage.

> The child can deal with hypothetical situations. . . Higher-order operations emerge in which abstract rules are used to solve problems. For example, algebra rather than a trial-and-error procedure is used to solve the problem:

"What number, if multiplied by 3 and reduced by 4, equals 20?" (p. 68)

Flavell (1992) questioned the Piagetian general stages stance that implies that the child has a characteristic mental structure that is applied to all content areas. Flavell countered by stating that contemporary developmentalists believe that cognitive development is more balanced with both general stagelike attributes and specific properties that relate to particular content areas. He explained that a child may function at a higher level in one content area than in another because of expertise acquired through extensive practice and experience.

While recognizing the contributions of Piaget, Siegler (1998) questioned the idea of explicit developmental stages. After explaining findings that demonstrated cognitive abilities in children younger than would be expected with Piaget's theory and findings that showed illogical thinking in adults, Siegler asserted that development generally is an incremental process that happens gradually over many years. He claimed that it is "increasingly indefensible. . .to state a single age at which children acquire a particular concept" (p. 331). Siegler also recognized the impact of existing knowledge on a student's ability to learn new information; "prior content knowledge influences what people learn as well as how much they learn" (p. 333).

Piaget believed that schemata are the cognitive or mental structures by which individuals adapt to and organize the environment (Wadsworth, 1984). Schemata never stop changing or becoming more refined. The child continually tries to make sense of his/her environment; he/she is continually constructing meaning. Assimilation is the cognitive process by which an individual integrates new knowledge into existing schemata. If a new stimulus cannot be assimilated, accommodation takes place. Accommodation involves either creating a new schema or modifying an existing schema. "*Equilibrium* is a state of balance between assimilation and accommodation. *Disequilibrium* is a state of imbalance between assimiliation and accommodation" (p. 17). In Piaget's theory, disequilibrium is the motivator that activates intellectual development.

Related Components from Bruner

Bruner's work represents one of the cognitive approaches to learning (Hohn, 1995). Hohn stated that "Bruner was responsible for the idea that learning is most meaningful to learners when they have the opportunity to discover relationships among concepts on their own" (p. 187).

A key point made by Bruner in his 1960 book, *The Process of Education*, is that early learning can make later learning more powerful and precise. He began with "the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). Bruner drew on Piaget's work with the stages of cognitive development to explain this ability to teach supposedly difficult concepts at an early age. Bruner focused on Piaget's concrete operational stage because this is the age for in-school learning. Bruner explained that an operation is a type of action that can be done by manipulating objects. According to Piaget's theory, children are not capable of dealing with abstract knowledge until they pass into the formal operation stage, between the ages of 10 and 14 (Bruner, 1960). "Later, at the appropriate stage of development and given a certain amount of practice in concrete operations, the time would be ripe for introducing them [students] to the necessary formalism" (Bruner, 1960, p. 38). In his introduction, Anglin (Bruner, 1973) summarized two of Bruner's contributions on instruction: The learner should be encouraged to be an active participant in the learning process and "curricula should be tailored to the learner's existing mode of representation" (p. xvi).

Bruner (1960) asserted that the transfer of learning is at the heart of the educational process. Transfer is what allows current learning to impact future applications. He explained that transfer is “dependent upon mastery of the structure of the subject matter” (p. 18). Mastering the important ideas of a field also involves the development of an inquiring attitude toward solving problems on one’s own. Bruner said, “To instill such attitudes by teaching requires something more than the mere presentation of fundamental ideas” (p. 20). Bruner conjectured that the appropriate attitudes would be developed through the “excitement about discovery” (p. 20). Thirteen years later, Bruner (1973) concluded that discovery aids in memory retrieval, “In sum, the very attitudes and activities that characterize figuring out or discovering things for oneself also seem to have the effect of making material more readily accessible in memory” (p. 412).

Unified Theory

An “emerging unified theory of educationally relevant learning” (Hohn, 1995, p. 402) combines both behavioral and cognitive approaches to learning. This integration of theories, when applied, results in educational practices which employ both cognitive and behavioral components. Thus, teaching techniques which utilize “the best of what each position has to offer” include both “reinforcement of new behavior and the learner’s own ability to process what is being presented” (Hohn, p. 403).

Goldin and Shteingold (2001) compared the behaviorist and constructivist perspectives in the context of mathematical representations. They pointed out the clash of these two differing philosophies in the public schools. Some educators “favor basic mathematical skills, correct answers through correct reasoning, individual drill and practice, more direct models of instruction, and measures of achievement through objective tests” (p.7). By comparison, they characterized the constructivist school of thought as including, among other items, children “making their own discoveries in mathematics, . . ., less use of teacher-centered models of instruction,. . . [and] group as well as individual problem-solving activity” (p.7). Goldin and Shteingold suggested the use of both schools of thought with an “inclusive educational philosophy—one that values skills and correct answers as well as complex problem solving and mathematical discovery, *without seeing these as contradictory*” (p.8).

Related Issues in Learning Mathematics

"Research on children's thinking has focused increasingly on the specific processes involved in learning" (Siegler, 1998, p. 284). The results of this research indicate that children either retrieve solutions from memory or revert to more time-consuming alternative strategies. Siegler compared his theory to the evolutionary aspect of biology; a better strategy eventually takes over and becomes the preferred strategy when solving mathematical problems. For example, young children solve the single-digit arithmetic problem of 3 + 6 with a variety of strategies including retrieval, counting on, counting on their fingers from one, or using related problems. With experience, children's strategies change; retrieval is the strategy that is increasingly utilized.

Retrieval is widely accepted as the dominant arithmetic strategy of adults (Resnick, 1989). Children with normal development move gradually toward the retrieval strategy between 7 and 11 or 12 years of age. Retrieval and other more sophisticated strategies are quicker and more accurate. Children tend to employ "the fastest approach that they can execute accurately" (p. 287). Accurately responding to problems reinforces the likelihood of producing the same correct answer in future instances.

Pushing children to use strategies which they are not ready to choose may retard learning (Siegler, 1998). An example Siegler described was forcing children to abandon counting on their fingers before they feel confident of their answers and have repeatedly reinforced the correct answers. "As is often the case, the most direct method for pursuing an instructional goal is not necessarily the most effective one" (p. 290).

Problems in learning mathematics have been attributed to a "combination of limited background knowledge, limited processing capacity, and limited conceptual understanding" (Siegler, 1998, p. 292). An example of limited conceptual understanding that Siegler offered dealt with the inversion principle which is the idea that adding and subtracting the same quantity to a number leaves the original number unchanged. Problems of the form $a+b-b=?$ (e.g., $6+9-9=?$) would be easily solved by a person who understands the inversion principle and the aligned fact that $+b-b=0$.

The superficial understanding of algebra by students who do well in algebra classes because they treat "the equations as exercises in symbol manipulation, without any connection to real-world contexts" (Siegler, 1998, p. 296) leads to misunderstanding. Some students make incorrect extensions of correct rules and may generalize the distributive principle that indicates $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$ and conclude that $a + (b \cdot c) = (a+b) \cdot (a+c)$.

Resnick (1989) asserted that children from families most alienated from schooling more often treat school mathematics as symbol manipulation. This is because these students are less likely to trust their own informal mathematics ideas since they do not have access to the more highly developed mathematical background knowledge that the mainstream culture provides to its children. Based on this assertion, "a general approach to school mathematics instruction that stressed concepts and explicitly engaged children's informally developed knowledge might be expected to yield particular benefits for minority children and perhaps for girls as well" (p.168).

Mayer (1999) cited studies that supported the idea that skills needed to solve mathematics problems can be taught thus enhancing the probability of a student's success regardless of his or her age. The answer to the question, "What does a student need to know to solve mathematics problems?" (p. 157) included four components. Students need linguistic and factual knowledge to translate the sentences of the problem, schematic knowledge in order to integrate the information into a coherent whole, strategic knowledge to devise a plan, and procedural knowledge to carry out the computations needed by the plan. An example where students learned these skills is the study where pre-algebra middle-school students experienced a 20-day program that emphasized daily opportunities to translate among relational sentences, tables, graphs, and equations. The students who participated in the program showed much improvement in comprehending and solving word problems as compared to the students in the control group. Another example involved first graders who were having difficulty with simple addition. In this case, training which emphasized the central conceptual structure of the number line enhanced the procedural knowledge for addition; the trained first graders exceeded by far the achievement of the control group of first graders. Mayer explained, "The learning of basic arithmetic procedures must be tied to the development of central conceptual structures in the child" (p. 200). The number-line training was a demonstration of the importance of helping students make the connection between procedures and concepts.

The Value of Manipulatives

Manipulative materials have been defined as “objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children” (Kennedy, 1986, p. 6). These objects can include materials such as balances which are specifically designed to address mathematical concepts, or environmental items such as measuring instruments and money. Kennedy explained that manipulatives became a focus of interest when student understanding, or meaning theory, surpassed the stimulus-response theories of the nineteenth and early twentieth centuries.

Hartshorn and Boren (1990) reviewed the research on using mathematical manipulatives for ERIC Digest. They found that manipulatives were useful in helping children move from the concrete level to the abstract level. A crucial component is the transition stage between these two levels. Teachers must carefully structure the use of the connecting or pictorial intermediate stage in order for students to make the connection.

Witzel, Smith, and Brownell (2001) advocated the use of the concrete-representation-abstract sequence. They specifically advocated hands-on experiences during the concrete phase; helping students understand abstraction on a concrete level involves the use of manipulatives. Then, at the representational phase, pictures help the students transition to the abstract, symbolic phase. Witzel et al. asserted that students with learning disabilities require this three-phase support in learning abstract mathematical concepts.

Professor Zemira Mevarech, head of the School of Education at Bar Ilan University, and formerly the chief scientist at the Israeli Education Ministry, cited the importance of guiding students from the concrete to abstract stages as they learn mathematics (Sa'ar, n.d.). In Sa'ar's article, Mevarech commented on the successful mathematics program from Singapore. Singapore eighth-grade mathematics achievement on the TIMSS report was ranked first in both 1995 and 1999 (Schmidt, 2000). Sa'ar quoted Professor Mevarech.

> "The Singaporean programs moves gradually," explains Mevarech, whose field of expertise is the teaching of mathematics, "from concrete illustration (objects) to visual illustrations (pictures) and then to abstract concepts - numbers. Thus a pupil deepens his understanding of the subject and internalizes the mathematical concepts." (¶ 25)

Moyer (2001) claimed that the sensory experiences with manipulatives help students understand mathematical concepts. Students have clearer mental images as a result of seeing and manipulating various objects (Kennedy, 1986). Mental imagery is crucial to the ability to think with numbers; imagery is the sensory-cognitive connection for mathematics (Bell & Tuley, 2006). Bell and Tuley explained that thinking mathematically requires the dual coding of language and imagery; the people who understand mathematics turn language into imagery. To emphasize the importance of imagery, they quoted Albert Einstein who said, "If I can't picture it, I can't understand it" (p. 1).

”Manipulative materials are significant learning aids in all four [of Piaget's cognitive] stages” (Kennedy, 1986, p. 6). Kennedy concluded that research “supports the use of manipulatives at all school levels” (p. 7). Middle level students need the manipulatives just as much as elementary students do since the middle level concepts are just as abstract to that age student as elementary concepts are to younger children. Stewart (2003) noted that the increase in abstraction in mathematics in elementary grades often coincides with the decrease in the use of manipulatives. As a result, few middle school teachers use manipulatives (Stewart, 2003). Much research exists on the importance of manipulatives at the elementary school level but there is little information involving manipulatives at the middle and high school levels (Weiss, 2006).

Some teachers may not have been trained to utilize the developments in cognitive psychology that suggest the value of manipulative materials (Weiss, 2006). Teachers who have traditional training may see themselves as the “expert who dispenses knowledge to students” (Moyer & Jones, 1998, p. 34). Those middle school teachers who have been instructed to use manipulatives must correctly implement them in order to benefit students in the middle grades (Weiss, 2006).

Moyer and Jones (1998) found variation in the implementation of manipulatives among middle school teachers. Teachers who do not value manipulatives as mathematical tools will not convey the usefulness of manipulatives to their students; students in these teachers’ classes may see the manipulatives as toys instead of tools. Conversely, “teachers who demonstrate how to use the manipulatives as tools for better understanding are opening doors for many students who struggle with abstract symbols” (p. 35). It is important to give students time for and access to manipulatives. With time, access, and the proper guidance, students will view manipulatives as necessary tools in the mathematical environment.

Testimonials attest to the contributions of manipulative materials. Allen (2003) quoted Johnny Lott, then president of

NCTM. “There are people in this country who think all kids can’t do mathematics, but we say that all kids can learn if math is presented in a good way—having a knowledgeable teacher and quality materials including texts and manipulatives” (p.1). Herbert (1985), a middle-school teacher, claimed that manipulatives are “good mathematics” because they result in better achievement, understanding, motivation, and student involvement.

Hands-On Equations Learning System

In an interview, Borenson explained the constructivist aspects of Hands-On Equations (Agency for Instructional Technology, 2003). "In HOE, we give the students a concrete representation of the algebraic symbols and algebraic processes. The symbols are represented by game pieces. The algebraic processes are represented by physical actions upon these pieces" (¶3).

The concepts learned with Hands-On Equations are basic and crucial such as the concept of equality (Borenson, 1994). Borenson also asserted that students learn the addition and subtraction properties of equality which reinforce their understanding of equality. Other concepts that are covered include the concept of a variable, and the addition and subtraction of positive and negative numbers. Students learn essential concepts related to zero, such as additive inverse and additive identity.

Research Studies on Hands-On Equations

Studies that involved Hands-On Equations are limited to four. A master's thesis was written by Barclay (1992) and another study was done by Leinenbach and Raymond (1996). Two dissertations were found. One was written by Busta (1993) and the other by Suh (2005).

The Leinenbach and Raymond (1996) contribution was a 2-year action research collaborative study where an eighth-grade teacher with 22 years of experience was confronted with the mandated dilemma of teaching algebra to all students. Concerned that she could not teach all students using the traditional approach, Leinenbach, the teacher, began teaching with a manipulative approach by using Hands-On Equations. After initiating this program, she became apprehensive about her students' transition to traditional algebra. She met with a university researcher, Raymond, who collaborated with Leinenbach on the action research project in order to find out if the Hands-On Equations manipulative approach would affect students' confidence and interest in algebra, affect students' ability to solve algebraic equations, and impact students' retention of algebraic skills beyond the eighth-grade experience.

Students were taught with a traditional method using the textbook during the first nine-weeks of the school year (Leinenbach & Raymond, 1996). All 26 lessons of Hands-On Equations were taught next. After the Hands-On Equations lessons were finished, the students were taught using the traditional textbook once again. In general, student scores or grades were higher during the manipulative phase than during either of the traditional phases. Student grades were lower in the second traditional textbook phase and caused Leinenbach to wonder if the students had been hampered by the manipulatives and were dependent on them in order to complete algebra problems. Another possibility Leinenbach considered was that the students were not making the connection between the concrete manipulatives and the abstract algebra in their textbooks. She surmised that a likely reason was that her students did not enjoy the textbook as much as the manipulatives and were less motivated to do well during the second exposure to the textbook.

When a standardized algebra test was administered to all eighth-grade students in Leinenbach's middle school, her students' performance far exceeded expectations (Leinenbach & Raymond, 1996). These results assuaged her fears and she concluded that her students had indeed bridged the gap between the concrete algebra and the abstract algebra required on the test. The reported conclusions from the first year's data indicated that most of the eighth-graders performed better academically with the manipulatives, Hands-On Equations, in comparison to the text. Students also expressed more positive attitudes about algebra when working with the manipulatives. The retention data from the second year of the study was not included in the publication.

In Barclay's (1992) study, 123 sixth graders were taught five lessons with Hands-On Equations. A pretest with 10 one-variable linear equations was administered. Students were asked to solve for x in equations such as $x + 6 = 9$,
$13 + x + x = x + 3 + 3x$, and $3(x + 1) = 10 + 2x$. Comparable posttests were given immediately after instruction, three weeks later, and six weeks later. Students used the Hands-On Equations manipulative materials during the tests. Scores, expressed as percents, were assigned to each student based on the number of correct responses. Thus a student who correctly answered 8 out of the 10 problems would receive a score of 80%.

The four tests were used to assess retention and concept mastery (Barclay, 1992). Only one student out of the total demonstrated mastery (80%) on the pretest; all but one student demonstrated mastery on the immediate posttest. Results on the three-week retention test were somewhat lower with an unexpected rise in scores on the six-week retention test. No intervening review of the algebra occurred between test administrations. Barclay carefully analyzed reasons for individual responses as she reported her data. She observed that many students entered the three-week retention test with “an overconfident, almost careless, attitude” (p. 30). Many students appeared eager for another opportunity to improve their scores with the six-week retention test. They also checked their work which was not done as frequently during the three-week testing. She concluded that these actions may account for the improvement seen in the six-week test scores.

Barclay (1992) decided that the students had indeed learned the algebra based on their test results. She reported that 100% of the students demonstrated at least 80% mastery on at least two of the three posttests and 87% of the students attained the 80% mastery level on all three posttests.

Busta (1993) studied the relationship between middle school students' knowledge of the concept of variable and the use of the concrete manipulatives Hands-On Equations. In addition, her study attempted to discern what effect, if any, the Hands-On Equations materials would have on students' attitudes toward mathematics. The research involved 13 teachers who volunteered to be a part of the study and 335 students at the sixth-, seventh-, and eighth-grade levels. Busta believed that all students in the study had limited exposure to the concept of variable since none of the textbooks prior to 1991 included this algebraic concept in the books the students were using. Students were pretested at the beginning of the school year and were posttested after seven weeks. The experimental group of students had one 30-minute Hands-On Equations lesson per week during the seven-week period. These lessons were from the first level of Borenson's Hands-On Equations materials.

The results of Busta's (1993) research revealed that students' attitudes were positively correlated with students' knowledge of the concept of variable. Students in the sixth grade who used Hands-On Equations did significantly better on the posttests than did students who did not learn with the manipulative materials. In the seventh grade group, all students increased their knowledge of the concept of variable with no difference between the experimental and control groups. The diverse differences in students in the eighth grade subjects led to unclear results for this group. There were no discernable differences in students' attitudes toward mathematics after the seven-week period. Busta recommended a qualitative study to better examine seventh graders' attitudes since the seventh-grade teachers indicated that their students had "thoroughly enjoyed working with the materials. This was not evident in the attitude measure results" (p. 118). Busta also recommended a follow-up study where all three levels of the Hands-On Equations materials would be taught.

Suh (2005) investigated the achievement of third graders when using virtual and physical manipulatives for adding fractions and balancing equations. The research was conducted with 36 students in two classrooms. One group of students worked with virtual manipulatives for four fraction lessons during one week and then had four lessons with the Hands-On Equations physical manipulatives during the second week of the study. Conversely, the other classroom worked with the fraction manipulatives during the first week and the virtual balance scale appelet during the second week of the study.

Students in the virtual manipulative fraction treatment group performed statistically better than the students who worked with the physical manipulative fraction circles. There was no statistically significant difference between the virtual and physical algebra methods. An examination of the algebra pretest and posttest revealed that the pictures used in the tests were of the virtual balance scale only and did not include pictures of the Hands-On Equations balance scale.

Advantages of Hands-On Equations

Borenson (Borenson and Associates, n.d.b) claimed to have taught third, fourth, and fifth graders to solve equations such as $4x + 2 = 3x + 9$ during live demonstrations at over 1000 workshops during a recent 10-year period. Systematic successful intervention with resulting mastery of basic algebra concepts is the goal espoused by Borenson (1994). Borenson also claimed that students actively engage in Hands-On Equations because the game-like format is interesting to them. In an interview with a newspaper reporter (Carnopis, 1987), Borenson claimed that Hands-On Equations gives confidence in algebra and builds the self-esteem that allows young students to feel good about algebra.

Ghazi (2000) observed Vicki Fisk's Somerset Elementary classroom in Maryland where Hands-On Equations was being taught. Ghazi reported that at that time these materials were a required part of the curriculum in Maryland's Montgomery County schools. During her observation, Ghazi noted the enthusiasm of 10-year-olds who wanted to explain how they solved $2(3x + 1) = x + 22$. The title and subtitle of her article, *Catch them young: Fear + loathing = algebra. Unless you're one of the thousands of 9-year-old Americans to have discovered that algebra = fun,* reflected her opinion after watching the children work with Hands-On Equations. After visiting American classrooms, Ghazi reported her findings in *The Guardian*, a United Kingdom publication, and wrote that British mathematics experts were astounded that such young children could work problems usually reserved for bright 12-year-olds or average 14-year-olds in Britain.

Memory and Long-Term Remembering

"Memory may be better understood when partitioned into separate components, stages, or processes than when treated as a unitary trait" (Terry, 2006, p. 225). Terry explained the need for this partitioning of memory. Generalizations about memory as a whole are not valid but descriptors of particular forms of memory are accurate. The various partitions of memory, "each with different characteristics, may be simpler to construct and to use than a single memory system that has many discordant facts" (p. 195).

The first partitioning of memory involves memory components (Terry, 2006). The two major components are short-term memory and long-term memory. Each component is considered to have opposing characteristics that distinguish the two components. Short-term memory is of a short duration lasting 15-30 seconds under laboratory testing conditions, has limited capacity of only a few items, and displaces the current contents with later-occurring items that results in the forgetting of the initial information. In contrast, long-term memory lasts indefinitely, is limitless in size, and is durable.

Long-term memory is further divided into three types of memory—procedural, semantic, and episodic (Tulving, 1985). Procedural learning is termed "knowing how" rather than "remembering that" which is associated with both episodic and semantic memory (Terry, 2006, p. 203). Terry separated procedural memory from the other two types but then further distinguished between episodic and semantic types of memory. Semantic memory is related to general knowledge whereas episodic memory includes autobiographical memories and is the personal memory system. Terry further explained that semantic memory is "more like dictionary or encyclopedic knowledge. It includes facts, words, language, and grammar" (p. 199). He claimed that the phrases "I remember" versus "I know" (p. 199) correspond to episodic and semantic memory, respectively.

A second partitioning of memory involves three stages of memory (Terry, 2006). Those stages are encoding, storage, and retrieval; any one of these stages could contain problems that lead to forgetting. For example, a student may not know the answer to a test question because he did not learn the material initially (encoding), he learned the answer but has lost it from memory (storage), or he cannot recall the answer (retrieval). Another example Terry offered suggested that memory lapses in older persons could be the result of weaker encoding rather than the assumed "faster forgetting or difficulty in retrieving memories" (p. 261). Effective retrieval from episodic memory depends on three general factors which include the distinctiveness of the memory, practice at retrieving the memory which may be in the form of taking repeated tests, and retrieval cues. One explanation for why distinctive events are retrieved better is that their retrieval cues are uniquely linked with a single memory. Cues that were encoded with the recalled item or event are good retrieval cues. The matching of cues between encoding and retrieval is termed encoding specificity.

A third partitioning of memory deals with processes of memory; "the kind or quality of processing determines memorability "(Terry, 2006, p. 210). The two sub-categories within this approach are depth of processing and transfer-appropriate processing. Depth of processing may be either shallow or deep and involves either maintenance rehearsal or elaborative rehearsal. Maintenance rehearsal, or the passive repetition of information, may contribute to memory loss. In contrast, elaborative rehearsal requires active processing by the learner on a deeper level. It is characterized by meaningful analysis and comprehension of the material. Learning strategies are examples of elaborative rehearsal. "Elaborative rehearsal should lead to longer retention than does maintenance rehearsal" (p 211). Terry added, "Depth-of-processing theory was so successful that, in a sense, it no longer exists as a separate approach to memory but has been assimilated into other approaches" (p. 212). The second sub-category is transfer-appropriate processing. This theory links the encoding and retrieval stages and asserts that the cognitive operations present at encoding need to match the cognitive thought processes in use at the retrieval stage for optimum remembering. Terry concluded his summary of the research on the three approaches to memory with a point of clarity. "The several approaches could be viewed as complementary rather than as exclusionary" (p. 214).

Bartlett's *Remembering: A Study in Experimental and Social Psychology* has been continually cited in the literature since it was first published in 1932 (Johnston, 2001). Johnston noted that Bartlett's book has been reissued twice, in 1964 and in 1995. Johnston also highlighted the "striking parallel between the treatment of F.C. Bartlett's theories of memory in the psychological literature and Bartlett's own characterization of reproductive memory as interest driven and constructive" (p.341). Bartlett (1932) described "every human cognitive reaction—perceiving, imaging, remembering, thinking and reasoning—*as an effort after meaning*" (p. 44).

Johnston (2001) linked Piaget and Bartlett by their common use of the schema concept. Johnston wrote that Bartlett's concept of the memory schema contributes to his continuing influence. Another aspect of Bartlett's contributions is his denouncing of the then-current popularity of Ebbinghaus and his emphasis on stimulus-response. Bartlett's viewpoint fed the "everyday-laboratory memory debate" (p. 344); Bartlett was interested in everyday responses from people rather than artificial laboratory settings for psychological research. Bartlett (1932) said, "I have used exactly the type of material that we have to deal with in daily life" (p. 204).

From numerous everyday-type experiments, Bartlett (1932) synthesized a theory of remembering that he described in his chapter ten. Even though Bartlett did not prefer the term schema, he settled on it as the term needed to describe his theory. " 'Schema' refers to an active organization of past reactions, or of past experiences, which must always be supposed to be operating in any well-adapted organic response" (p. 201). Bartlett went on to say that "remembering appears to be far more decisively an affair of construction rather than one of mere reproduction" (p.205). He also explained that attitude impacts all of memory.

Several variables on long-term memory for knowledge learned in classrooms were delineated by Semb, Ellis, and Araujo (1993). They included the "degree of original learning, the tasks to be learned, characteristics of the retention interval, the method of instruction, the manner in which memory is tested, and individual differences" (p.305). In three experiments with college students, these researchers found that students remembered a great deal of what they learned in college. The research results indicated that all listed variables do impact memory.

Howe (2000) drew on the work from classic literature on memory retention in order to conclude that “slower learners may forget more rapidly” (p. 122). He thus asserted that it is necessary to know the amount of learning that was acquired initially in order to interpret the outcomes of later assessments of long-term retention. Howe claimed that without information about the state of learning at the end of acquisition, the confounding of learning and forgetting is practically guaranteed.

"The primary determinant of persistent retention is the initial level of acquisition" (Terry, 2006, p.302). Students who took higher-level courses in high school and earned better grades retained more. Terry cited studies where most of the forgetting occurred in the first few years; the retained material then remained stable for as many as 50 years afterward.

Bahrick, Bahrick, and Wittlinger (1975) reported on 50 years of memory for names and faces. These authors made the case for the non-laboratory approach to the study of memory. They found that the research participants thoroughly remembered classmates by name or face recognition for 14 years after graduation and that memory did not decline noticeably until 47 years later. Free recall tests ranged from 15 percent recall for recent graduates to less than 7 percent recall for the oldest graduates. "This is far better than would be anticipated from laboratory studies" (Terry, 2006, p. 301). Bahrick et al. concluded that social context is important to recall performance and less valuable with recognition performance. These researchers attributed the very slow forgetting to the effects of distribution of practice and overlearning (Bahrick et al.). The repeated exposure to classmates and the spaced repetition of these exposures over holidays and summer vacations made the difference in this study of naturalistically learned material (Bahrick et al.).

Bahrick (1984) also conducted a study that dealt with knowledge learned in school, specifically the Spanish language. In this study, 773 individuals were tested. A noticeable decline in retention occurred in the first 3-6 years but thereafter remained unchanged for periods of up to 30 years before once again declining. The research data revealed that rehearsal did not occur enough to be a factor in this study. This fact led to "significant conclusions regarding the semipermanent nature of unrehearsed knowledge" (p. 2). Remembering was a function of the level of original training, the grades received in Spanish courses, and the method of testing (recall vs. recognition).

Semb, Ellis, and Araujo (1993) asked, "How much is lost from classroom learning over periods typically much longer than those studied in laboratory experiments?" (p. 306). The authors mentioned the earlier laboratory studies of Ebbinghaus and others which found that memory should decline over time. Semb et al. discovered that students retained a great deal of the original learning. The child psychology college students in the study remembered 85% after 4 months and about 80% after 11 months. The results supported the theory that the degree of original learning is a definite factor in how much is remembered.

Groups of college students who had completed a cognitive psychology course were studied (Conway, Cohen, & Stanhope, 1991). The largest amount of forgetting happened within 3 to 4 years. In the very long-term retention of cognitive psychology, concepts were remembered better than proper names of theorists. General factual knowledge and knowledge of research methods showed no decline. The authors concluded that detailed and specific information may be retained in memory over very long retention periods.

Some researchers investigated unusual retention rates. This phenomenon is termed hypermnesia and is defined as the "abnormally vivid or complete memory or recall of the past" (Woolf, 1981, p.558). In the context of looking at the effect of repeated testing on students' memory, Bahrick and Hall (1993) found that hypermnesia is present even when testing intervals are long. This occurred when the tests covered a stable body of content knowledge.

Self-Efficacy and Mathematics

Bandura (1986) defined self-efficacy as the self assessment of one's capability to succeed to a certain level in specific subject areas. With this definition, a person's self-efficacy is "expected to vary depending on the particular activity domain or situation under consideration" (Lent, Brown, & Gore, 1997, p.307). Hohn (1995) clarified the specificity of this concept with an explanation and example, "Self-efficacy judgments are specific to certain domains in which we judge ourselves to possess competence. We describe ourselves, for example, as 'good in writing but not so good with numbers'" (p. 168). Pajares and Miller (1995) confirmed the importance of matching the self-efficacy assessment with the particular task.

They warned against the use of global statements such as “I am not so good at mathematics” (Pajares & Miller, 1995, p.197).

Four major sources used by people to develop their self-efficacy beliefs are personal performance accomplishments, vicarious learning, verbal persuasion, and emotional arousal (Bandura, 1986). Lent, Lopez, and Bieschke (1991) found that personal performance accomplishments contributed the most influence as a source of efficacy information.

The applied implications of the research of Lent, Lopez, and Bieschke (1991) included educational interventions for students who were “unrealistically low in mathematics self-efficacy” (p. 429). The suggested interventions included systematically structured mastery experiences in mathematics. Lent et al. stated that students need enough exposure to mathematics to form an opinion about themselves. They suggested that numerous successful structured mastery experiences would contribute to a more positive mathematics self-efficacy in students.

Self-efficacy may impact important outcomes such as career choice (Hackett & Betz, 1989), and effort and persistence in the face of obstacles (Bandura, 1986). A study that focused on the relationship between interests and self-efficacy as predictors of mathematics/science-based careers was conducted by Borget and Gilroy (1994). Their results showed that interest is the stronger indicator of career choice but self-efficacy is the better predictor of success and persistence in the chosen career (Borget & Gilroy, 1994). Lent, Lopez, and Bieschke (1991) found that a pattern of past success experiences possibly promotes self-efficacy which then leads to an increased interest in the particular domain. Lent et al.(1991) explained that this interest then motivates further exposure to the subject matter which then impacts future career choices.

More recent research (Pietsch, Walker, & Chapman, 2003) confirmed that self-efficacy in mathematics predicts future performance in mathematics. The work of Marsh, Dowson, Pietsch, and Walker (2004) reexamined and questioned the relations among self-efficacy, self-concept, and achievement. The work that they reexamined was that done by Pietsch, et al. in 2003. The reexamination did not dispute the idea that self-efficacy impacts mathematics performance.

A study involving 518 fifth- and seventh-grade children tried to discover why girls outperform boys in terms of grades. This research focused on the differing ways boys and girls approach schoolwork. The results indicated that sex differences in disruptive classroom behavior and in achievement goals influence the students' grades (Kenney-Benson, Pomerantz, Ryan, & Patrick, 2006). This advantage for girls over boys did not continue with achievement tests.

> Although girls should feel more efficacious than boys because of their better performance, girls often have lower self-efficacy in stereotypically masculine areas, such as math, than do boys. The current findings suggest that in the context of taking achievement tests, girls' self-efficacy may negate some of the positive effects of how they approach school. (p.22)

Attitude Toward Mathematics

Mager (1968) described attitude as a general tendency of a person to act in a “certain way under certain circumstances” (p. 14). A person with a favorable attitude exhibits some sort of *moving toward* behavior by which one predicts continued “moving toward” behavior. Conversely, avoidance behavior is associated with a negative attitude. Mager asserted that teachers should influence students to develop a favorable attitude toward a subject in order to maximize the possibility of remembering, using, and learning more about that subject in the future.

Neale (1969) defined attitude toward mathematics in terms of the characteristics delineated in inventories that were used to measure it. Those characteristics included "a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless" (p. 632). On the basis of studies he reviewed, Neale first hypothesized that students develop increasingly unfavorable attitude toward mathematics as they progress through school and secondly, the impact of those negative attitudes on the learning of mathematics is limited. He stated that the first hypothesis would be of more concern if the second hypothesis were not true.

Ma and Kishor (1997) conducted a meta-analysis on the relationship between attitude toward mathematics (ATM) and achievement in mathematics (AIM). These researchers were prompted to do their study because of the lack of consensus in the research literature about this potential relationship. The criteria for the inclusion of each of the 113 studies employed a definition of ATM that was similar to the one used in the meta-analysis, an investigation of the relationship between ATM and AIM, the use of psychometrically-developed instruments, no inclusion of experimental interventions on either attitude or achievement, elementary and/or secondary students, and enough detailed data to calculate an effect size. The overall mean effect size was 0.12 which was "statistically significant but not strong for educational practice" (p. 39). The effect size for the causal relationship for ATM(cause) and AIM (effect) was 0.08 and deemed to have no practical implication.

One conclusion drawn by Ma and Kishor (1997) was that current attitude measures are "very crude approximations to 'true' attitudes" (p. 39). They suggested that researchers should refine these assessment tools. They attributed weak relationships at the elementary school level to younger students' inability to verbalize their attitude toward mathematics and the lack of stability at that age level. Ma and Kishor's review of the meta-analysis results caused them to believe that the ATM-AIM relationship may be impacted the most during the junior high school years. One of their suggestions for further research on the ATM-AIM relationship was to include mathematics ability as a key variable.

Moyer and Jones (1998) claimed that the use of manipulatives has "the potential to improve student attitudes and student intrinsic motivation" (p. 35). Students who were allowed to use manipulative materials in their daily mathematics lessons appreciated the usefulness of these learning tools for constructing meaning. Having access to manipulatives on individual desks supported this improved attitude by giving the students more time to explore, investigate, and construct meaning. Moyer and Jones advocated the use of manipulatives as often as other mathematical tools such as rulers and protractors.

Singh, Granville, and Dika (2002) found that mathematical achievement was affected by academic time, mathematics attitude, and motivation. Three items were used to gather data about the attitude construct. Respondents were asked about whether or not they looked forward to mathematics class, the usefulness of mathematics in the future, and the student's level of boredom in school. While academic time exerted the strongest direct effect, mathematics attitude affected both academic time and achievement. Their research concluded that attitude is influential in explaining mathematics achievement variations.

Summary

Algebra is distinctly different from arithmetic; algebra focuses on general relationships and has been termed the generalization of arithmetic (Costello, 1993; Esty, 1999; MacGregor & Stacey, 1999; Saul, 2001; Tierney & Nemirovsky, 1997). Children as young as sixth grade are able to go from the specific to the general by discerning patterns and then generating helpful formulas (Herbert & Brown, 1997).

The National Assessment of Educational Progress (NAEP) assessed important, informal algebraic concepts related to patterns and relationships (Kenney & Silver, 1997). The 1992 NAEP assessed fourth graders' algebraic thinking involving patterns of figures, symbols, or numbers. Fourth graders could reason with simple patterns but had more trouble with complex patterns and explaining their mathematical reasoning about patterns. A close examination of the NAEP algebra and functions question data for fourth-grade students from *The Nation's Report Card* (National Center for Education Statistics, n.d.) revealed that a majority of the students could answer procedural knowledge questions correctly but stumbled when required to deal with conceptual-understanding and problem-solving questions. This information confirms the pattern for American students of performing well on computational tasks but not achieving as well on problems that demand deeper mathematical understanding (Kilpatrick, Swafford, & Findell, 2001).

Data from the state of Kansas (Center for Educational Testing and Evaluation, n.d.a; Center for Educational Testing and Evaluation, n.d.b) reported achievement levels of fourth-grade students. These data indicated an improvement in algebraic knowledge for Kansas fourth graders from 2000 to 2005.

The *Third International Mathematics and Science Study* (TIMSS) was a large-scale international study that revealed the comparative inadequacies of algebra learning in American schools (U.S. National Research Center, 1996). For example, United States students in the eighth grade study arithmetic, fractions, and a small amount of algebra in contrast to both Japan and Germany whose students receive thorough exposure to both algebra and geometry.

Kieran (1992) included two areas that impact student learning in algebra: the content and the students. Various authors make the case that the content of algebra is inherently difficult because algebra is both a language and an abstract system with specific rules that are difficult to learn (Esty, 1999; Hatfield, Edwards, Bitter, & Morrow, 2005; Usiskin, 1996; Von Rotz & Burns, 2002). Kieran (1989) observed that the language aspect of algebra is difficult for high school students to decipher. Kieran's (1992) analysis of the research related to the learning of algebra supported her overall conclusion that students do not understand the more difficult structural aspects of algebra. Bruner (1960) and Rittle-Johnson and Alibali (1999) also emphasized the importance of helping students understand the structure of the subject.

Kilpatrick et al. (2001) confirmed that many students have difficulty in making the transition from school arithmetic to school algebra for various reasons including the symbolism of algebra. Rubenstein and Thompson (2001) asserted that students who do not master the standard symbolism of mathematics will be hindered at some point in their mathematical careers. Oftentimes, students do not understand the equality symbol as an indicator of equality (Behr, Erlwanger, & Nichols, 1976; Erlwanger & Berlanger, 1983; Kieran, 1981; Saenz-Ludlow & Walgamuth, 1998).

A lack of opportunity to learn algebra may be another real issue for students; the NAEP (National Center for Education Statistics, n.d.) data bank revealed how infrequently fourth-grade teachers addressed algebra and functions. When teachers do teach the subject, the Third International Mathematics and Science Study (TIMSS) revealed that students often experience traditional teaching methodologies rather than the reform curriculum suggested by research (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). Carraher, Schliemann, Brizuela, and Earnest (2006) concluded that students' difficulties with algebra may more closely relate to teaching techniques rather than a developmental inability. One reason for this lag in appropriate teaching may be that United States mathematics teachers rarely have time to teach any subject in depth because they are expected to teach such a wide range of subjects (Lane, 1996). This breadth instead of depth is obvious in the textbooks and curriculum and makes a difference in student learning (Schmidt, 1996). An often-quoted accusation described the United State's curriculum, textbooks, and teaching as "a mile wide and an inch deep" (Schmidt, 1996, ¶ 5).

Two distinct theories over the past 100 years have shaped the discussion on learning. Behaviorism deals with externally observable events, instructional manipulations and outcome performance, whereas a cognitive approach to learning includes internal factors such as learning processes and existing learner characteristics (Mayer, 1999). An example of behaviorism in mathematics education is the use of flash cards when teaching math facts (Hohn, 1995). Constructivism is a cognitive learning theory that emphasizes that understanding must be constructed by the learner (Lemlech, 2002). Constructivism has become the accepted theoretical position among mathematics education researchers (Battista, 1999; von Glasersfeld, 1995). Others (Hohn, 1995; Goldin & Shteingold, 2001) advocated for a unified theory that includes both schools of thought. Such an inclusive educational philosophy would include both behaviorism and constructivism and would value both skills acquisition and complex problem solving (Goldin & Shteingold, 2001).

When learning mathematics, children either retrieve solutions from memory or revert to more time-consuming alternative strategies that make sense to them (Siegler, 1998). Attempted recall with limited conceptual understanding leads to problems in the learning of mathematics. In algebra, Siegler asserted that superficial understanding is exemplified by students who merely manipulate the algebraic symbols without understanding any real-world applications. Such students may make incorrect extensions of correct rules and generalize inaccurately. Current thinking on the cognitive development of 11- and 12-year-olds questions Piaget's idea of explicit general developmental stages (Flavell, 1992; Siegler, 1998).

Contemporary developmentalists believe that cognitive development is more balanced with both general stagelike attributes and specific properties that relate to particular content areas (Flavell, 1992). Prior content knowledge influences what people are able to learn (Siegler, 1998). Mayer (1999) and Bruner (1960) supported the idea that skills needed to solve mathematics problems can be taught regardless of a student's age.

Manipulative materials are objects that can be handled by the learner (Kennedy, 1986). Manipulatives have been shown to help children move from the concrete level to the abstract level (Hartshorn & Boren, 1990). Hartshorn and Boren found that a transition stage between these two levels is crucial. Teachers must carefully structure the use of the connecting or pictorial intermediate stage in order for students to make the connection (Hartshorn & Boren, 1990; Sa'ar, n.d.; Witzel, Smith, & Brownell, 2001). Mental imagery formed by handling manipulative materials helps students understand mathematical concepts (Bell & Tuley, 2006; Kennedy, 1986; Moyer, 2001).

Manipulative materials should be used at all school levels (Kennedy, 1986). Stewart (2003) noted that the increase in abstraction in mathematics in elementary grades often coincides with the decrease in the use of manipulatives. Much research exists on the importance of manipulatives at the elementary school level but there is little information involving manipulatives at the middle and high school levels (Weiss, 2006).

Henry Borenson (1994) created manipulative materials, the *Hands-On Equations Learning System*, to support the learning of algebra for students as young as 8 years old. Borenson claimed that the system (hereinafter referred to as Hands-On Equations) imparts important mathematical content, promotes mathematical interest, and heightens student self-esteem. These materials were specifically designed to meet the algebraic needs of teachers and students in the elementary grades. Borenson's materials have been available since the early 1990s, but few research studies have been done to explore the value of Hands-On Equations.

Four studies dealt with Hands-On Equations. The 123 sixth graders in Barclay's (1992) study were taught five lessons with Hands-On Equations. Posttest results showed that 100% of the students demonstrated at least 80% mastery on at least two of three posttests. Leinenbach and Raymond (1996) worked with eighth graders. When these students took a standardized algebra test, their performance far exceeded expectations. The students also expressed more positive attitudes about algebra when working with the manipulative materials in Hands-On Equations. Busta (1993) studied the impact of Hands-On Equations on 335 middle school students in grades 6, 7, and 8. The students were taught one lesson per week for seven weeks. The sixth graders who experienced Hands-On Equations did significantly better on the posttest than the control group did. Virtual and physical manipulatives for adding fractions and balancing equations were included in Suh's (2005) research. Two classrooms with a total of 36 students were taught four lessons with virtual manipulatives in one content area (fractions or algebra) and then experienced four lessons with physical manipulatives on the other topic (fractions or algebra). The physical manipulatives used when teaching the balancing of equations was Hands-On Equations. Students in the virtual manipulative fraction treatment group performed statistically better than the students who worked with the physical manipulative fraction circles. There was no statistically significant difference between the virtual and physical algebra methods.

The advantages of Hands-On Equations have been reported by others (Borenson and Associates, n.d.b; Carnopis, 1987;

Ghazi, 2000). Borenson claimed that third, fourth, and fifth graders were taught to solve equations such as $4x+2=3x+9$ during live demonstrations at over 1000 workshops during a recent 10-year period. He explained that the game-like format is interesting to students (Borenson, 1994) and gives them confidence and self-esteem that allows them to feel good about algebra (Carnopis, 1987). Ghazi (2000) observed and reported on the enthusiasm of 10-year-olds when working with Hands-On Equations. She wrote that British mathematics experts were astounded that such young children could work problems usually reserved for bright 12-year-olds or average 14-year-olds in Britain.

Research has shown that memory and learning are related topics. Memory is better understood when partitioned rather than considered as one unit (Terry, 2006). Terry explained that generalizations about memory as a whole are not valid but descriptors of particular forms of memory are accurate. One partitioning involves two major memory components: short-term memory and long-term memory. Long-term memory is further divided into three types of memory—procedural, semantic, and episodic (Tulving, 1985). A second partitioning of memory involves three stages of memory (Terry, 2006). Those stages are encoding, storage, and retrieval; any one of these stages could contain problems that lead to forgetting. Effective retrieval from episodic memory, the autobiographical or personal memory system, depends on factors such as the distinctiveness of the memory and retrieval cues. One explanation for why distinctive events are retrieved better is that their retrieval cues are uniquely linked with a single memory. Cues that were encoded with the recalled item or event are good retrieval cues. A third partitioning of memory deals with processes of memory and has two sub-categories, depth of processing and transfer-appropriate processing (Terry, 2006). The importance of depth of processing is widely accepted as leading to comprehension of material. Transfer-appropriate processing links the encoding and retrieval stages for optimum remembering. Terry emphasized that the three approaches to memory were complementary rather than exclusionary.

Bartlett's *Remembering: A Study in Experimental and Social Psychology* has been continually cited in the literature since it was first published in 1932 (Johnston, 2001). Bartlett (1932) described "every human cognitive reaction—perceiving, imaging, remembering, thinking and reasoning—*as an effort after meaning*" (p. 44). Johnston (2001) linked Piaget and

Bartlett by their common use of the schema concept. Schema involves the active organization of past reactions or past experiences (Bartlett, 1932). Bartlett asserted that remembering is more construction than mere reproduction. He also explained that attitude impacts all of memory.

Several variables on long-term memory for knowledge learned in classrooms were delineated by Semb, Ellis, and Araujo (1993). They included the "degree of original learning, the tasks to be learned, characteristics of the retention interval, the method of instruction, the manner in which memory is tested, and individual differences" (p. 305). Howe(2000) emphasized the importance of knowing the initial degree of learning in order to interpret later assessments of long-term retention. Terry (2006) stated that the initial level of acquisition impacts retention. Bahrick, Bahrick, and Wittlinger (1975) made the case for the non-laboratory approach to the study of memory. They found that people could remember classmates' names and faces for almost 50 years. They attributed this long recall to distribution of practice and overlearning of the naturalistically learned material. Bahrick (1984) looked at knowledge learned in school and found that there is a semi-permanent nature of unrehearsed knowledge. Other researchers (Conway, Cohen, &

Stanhope, 1991; Semb, Ellis, & Araujo, 1993) also found that students remembered classroom learning over long periods of time.

Hypermnesia is the "abnormally vivid or complete memory or recall of the past" (Woolf, 1981, p. 558). Bahrick and Hall (1993) concluded that hypermnesia may be apparent when assessment tests cover a stable body of content knowledge.

Research has studied the role of self-efficacy on students' learning. Bandura (1986) defined self-efficacy as the self assessment of one's capability to succeed to a certain level in specific subject areas. Hohn (1995) and Pajares and Miller (1995) also explained that self-efficacy is specific to particular tasks or certain domains. Of the major sources of developing self-efficacy, personal performance accomplishments contributed the most influence (Lent, Lopez, & Bieschke, 1991). Lent et al. recommended educational interventions for students with low mathematics self-efficacy. They suggested that numerous successful structured mastery experiences would contribute to a more positive mathematics self-efficacy in students. Self-efficacy may impact important outcomes such as career choice (Hackett & Betz, 1989), and effort and persistence in the face of obstacles (Bandura, 1986; Borget & Gilroy, 1994). More recent research (Pietsch, Walker, & Chapman, 2003) confirmed that self-efficacy in mathematics predicts future performance in mathematics.

Many researchers have studied the impact of attitude on student learning of mathematics. A favorable attitude toward mathematics would lead to moving toward behavior whereas avoidance behavior is associated with a negative attitude (Mager, 1968). Mager asserted that teachers should influence students to develop a favorable attitude toward a subject in order to maximize the possibility of remembering, using, and learning more about that subject in the future.

Ma and Kishor (1997) conducted a meta-analysis on the relationship between attitude toward mathematics (ATM) and achievement in mathematics (AIM). The overall mean effect was "statistically significant but not strong for educational practice" (p. 39) and the effect size for the causal relationship for ATM (cause) and AIM (effect) was insignificant and deemed to have no practical implication. They concluded that current attitude measures do not reflect true attitudes and recommended that researchers should refine these assessment tools for better future results. Ma and Kishor also believed that the ATM-AIM relationship may be impacted the most during the junior high school years. Singh, Granville, and Dika (2002) found that attitude is influential in explaining mathematics achievement variations.

Moyer and Jones (1998) claimed that the use of manipulatives has "the potential to improve student attitudes and student intrinsic motivation" (p. 35). Students used manipulatives as learning tools for constructing meaning. Moyer and Jones advocated the use of manipulatives as often as other mathematical tools such as rulers and protractors.

CHAPTER 3

THE RESEARCH DESIGN

Introduction

This study was designed to examine the perceptions of high school graduates who experienced the mathematical materials from Hands-On Equations when the students were in the sixth grade in 1997. As a comparison, the investigation also included the perceptions of students who did not experience Hands-On Equations during their 1997 sixth-grade year. Four research questions were addressed.

1. For the students who experienced Hands-On Equations, what is the perceived value of these materials?
2. Did the Hands-On Equations lessons create student perceived differences in subsequent learning in algebra classes for students taught with Hands-On Equations?
3. Is there a difference in present mathematics self-efficacy between students taught with Hands-On Equations and those who did not experience these teaching materials?

4. Are there other differences related to (a) attitudes toward mathematics, (b) student achievement in mathematics, and (c) student ability to solve simple linear equations between students taught with Hands-On Equations and students who were not?

Three components are relevant to this study: the 1997 work with the sixth-grade classroom and the action research study conducted at that time, the design of the current study, and the pilot interview that was conducted in order to evaluate the feasibility of the research design.

This chapter first describes the Hands-On Equations materials. This is followed by a description of the 1997 experience with these materials. The chapter then presents the design of the study including a description of the participants, the data collection procedures that were employed in the study, and the methods used for data analysis. Finally, this chapter documents the outcome of a preliminary interview conducted as a pilot study.

Hands-On Equations

The *Hands-On Equations Learning System* is the full name for the materials referred to as Hands-On Equations in this study; Henry Borenson is the creator of these copyrighted materials. Hands-On Equations makes algebraic concepts concrete and thus attainable for all in grades three to adult (Borenson, 1994). Hands-On Equations is a set of instructional materials that includes student and teacher manipulative materials, three levels of teacher manuals that explain the 26 lessons, and worksheets for each lesson. The system "provides concrete and invaluable experience in using the basic ideas associated with algebraic linear equations" (Borenson, 1994, p. 23). Borenson (1994) claimed that the system imparts important mathematical content, promotes mathematical interest, and heightens students' self-perceptions as learners.

Borenson (1994) asserted that some of the understandings that may be attained by students using these materials are the concepts of a balanced equation, variables, and essential concepts related to zero including the additive inverse and the additive identity. The scope of the lessons is described in a summary of the lesson objectives for each of the 26 lessons included in Hands-On Equations; the summary is included in Appendix A.

Borenson's web site, borenson.com, includes links to a brochure, a sample problem, a video demo, photos, a price list, an order form, a current seminar schedule, verbal problems, program validation, an interview with Dr. Borenson, and contact information. The “brochure” (Borenson and Associates, n.d.a) and “sample problem” (Borenson and Associates, n.d.c) links describe the materials. The teacher manual that accompanies Hands-On Equations also describes the materials and procedures (Borenson, 1994). During Hands-On Equations lessons, students are actively engaged in a game-like atmosphere where “legal moves” dictate the moving of pawns and cubes on an individual mat that pictures a balance scale. Comparable teacher materials that are slightly larger are used to model the various lessons. The teacher balance scale is a stationary, plastic balance scale on which both blue and white pawns and red and green number cubes may be placed. The blue pawns represent the variable x and the white pawns are called “star” until it is revealed that they represent $(-x)$. Red and green integer number cubes represent positive and negative integers used as constants. These pawns and cubes are manipulated to solve simple linear, one-variable equations such as $2(x + 4) + x = 2x + 10$ and $2x + (-x) + 3 = 2(-x) + 15$. By Lesson 7, students are able to solve for the unknown x in equations such as $2(x + 4) + x = x + 16$ (Dvorak, 1988).

1997 Experience with Hands-On Equations

In January 1997, this researcher taught Hands-On Equations to a class of 23 sixth graders. Algebra plus problem solving was the focus of that month in this classroom in Baldwin Junior High School. A 45-minute lesson was taught on most days starting January 4, 1997. The first 18 class sessions followed the explicit guidelines provided in the Hands-On Equations teacher manual. Sample pages from the teacher manual are provided in Appendix B. A summary that identifies the concepts that were covered in the first 18 lessons during January 1997 is found in Appendix A. The last 3 class sessions linked algebraic problem solving with the Hands-On Equations methods. Hands-On Equations only deals with problem solving incidentally; that component was added because this researcher/teacher believes that the ability to solve real problems is the ultimate purpose of studying mathematics. An action research question to be answered was, "Would algebraic thinking aid elementary-aged students in problem solving?" A test that was used as both the pretest and posttest and that required problem solving was compiled from resources included with Hands-On Equations. A copy of that test is in Appendix C.

The students were able to build on the algebraic ideas and terminology when the problem-solving aspects were introduced later in the month. While only 3 of the 21 class sessions with the students specifically dealt with problem solving, students were able to employ their newly-learned Hands-On Equations information when solving problems. Armed with a vocabulary and an intuitive, conceptual understanding of terms such as "equals" and "variables," the students were given word problems that could be answered by creating and solving one-variable equations. The students were able to discern what the questions were asking for, or the unknowns. The link to, and support for problem solving, came when the class, with teacher guidance, devised a problem-solving sequence that incorporated their new algebraic knowledge. This sequence worked well for this group of sixth graders and was visible on the board when they took the posttest.

1. Read the problem.
2. Label the unknown (x).
3. Set it up.
4. Write the equation.
5. Solve the problem.

   Find the unknown (x).

6. Write down the answer.

   $x = ?$

7. Check your work.

   Write down the check.

Participants in the 1997 Study

The sixth-grade classroom in which this study occurred had 23 student members; none were minority students. Of these 23 students, 13 were boys and 10 were girls. Special services were provided to 7 students; 2 of the boys were identified as gifted and 3 boys and 2 girls were identified as learning disabled. None of these 7 students who were identified for special services were out of the room during the Hands-On Equations lessons. The teacher labeled 2 students as low-achieving with no official designation or special services provided to them.

Analysis of the data was conducted for only 18 of these 23 students since only 18 had both pretest and posttest scores for various reasons. Of these 18 students, 11 were boys and 7 were girls. The 7 students who received special services were included in these 18 students. The 2 low-achieving students who were not identified as needing special services were also included in the 18 who had both a pretest and a posttest.

Measurement for the 1997 Study

The specific attribute of interest in this action research study was problem-solving ability as affected by the learning of the Hands-On Equations approach to foundational algebraic concepts. This cognitive attribute was defined as the ability to answer correctly problems that were administered in pretest and posttest settings. Gall, Gall, and Borg (2003) explained that “the one-group pretest-posttest design is especially appropriate when you are attempting to change a characteristic that is very stable or resistant to change” (p. 391). This design works when “the likelihood that extraneous factors account for the change is small” (p. 391). The sixth graders had no other classroom exposure to algebra or algebraic problem solving between test dates. It was highly unlikely that the students learned algebra or algebraic problem solving in any other way during the interim between test dates. Thus the learning of algebra and algebraic problem solving qualified as resistant to change. Maturation over this short amount of time did not likely affect the improvement of students’ algebraic problem-solving skills.

A seven-question, word-problem test was compiled from Hands-On Equations Verbal Problems Sides A and B (Borenson, 1994) to measure student achievement and to use as both the pretest and the posttest. The same form of the test was used because there were at least six weeks between test administrations. Also, the students did not see the test results from the pretest and thus had no opportunity to study the pretest before taking the posttest. The students had an awareness of the purpose of the pretest in relation to general problem solving but had no way to link the pretest with the Hands-On Equations procedures that were subsequently taught to them. The students recognized the alignment of the posttest with Hands-On Equations and the problem-solving work that had transpired in class prior to the posttest.

The pretest was administered on January 2, 1997. A first posttest was done on February 14, 1997, and the last posttest was given on April 4, 1997. An ethical concern arose during the administration of the first posttest. This posttest was given immediately after the completion of the 21 teaching sessions on February 14, 1997. Recognizing the diligent work of her students during the preceding six weeks, the classroom teacher, with good intentions, helped several students during the posttest thus invalidating the outcomes of their tests. Since both the researcher/teacher and the classroom teacher were present during all of the teaching sessions and during the posttest, it was obvious that her interest was in helping each child do as well as possible, thinking that a little bit of additional information might allow them to display all of their new algebraic knowledge. These actions tainted the posttest results. Consequently it was decided to retest the students after a period of time had elapsed. The second administration of the posttest was on April 4, seven weeks after the first attempt. The data from the January 2 pretest and the April 4 posttest are reported.

Since this researcher/teacher was an invited guest in the sixth-grade class, the results gathered were not used for high-stakes decisions. The classroom teacher recorded daily homework from the work in January and February as she would any other assignments in the mathematics curriculum. Student results from the pretests and posttests contributed to diagnosis and formative judgments about the students but did not impact placement decisions or summative grading.

The 1974 Family Education Rights and Privacy Act or FERPA (the Buckley Amendment) spells out student privacy rights. In accordance with FERPA, none of the data collected has or will be linked with any particular student in a way that would allow anyone to discern the identity of the student.

Students were treated with dignity and were encouraged to take risks in a safe, supportive classroom climate that promoted student success. Psychologically, students appeared to enjoy the Hands-On Equations sessions and learned as evidenced by their daily work and the posttest results.

Test results. While 23 students made up the sixth-grade class, 18 students had both pretest and posttest scores. Some students were absent on test dates for various reasons. The results of this 1997 study included the data from the 18 students who had paired sets of scores. The data are located in Appendix D.

The pretest had a positively skewed distribution of scores. While it was not the intent of the test to discriminate among students, the positively skewed pretest made obvious two of the more talented students who scored a 5 and a 6. No student achieved a perfect score of 7 on the pretest. The brighter students in the class scored above the pretest mean of 2.39; 8 of the 18 students scored 3 or above on the pretest. The pretest median was 2 with a mode of 1. The range on the pretest was 5 with scores falling between 1 and 6.

The posttest had a negatively skewed distribution. The negatively skewed posttest results permitted the discernment of those students still struggling with problem solving as determined by this test. The posttest mean was 4.83. The posttest median was 5 with a mode of 5. The range on the posttest was 5 with scores falling between 2 and 7.

Table 1 shows the comparisons of the descriptive statistics. The pretest mean was 2.39 and the posttest mean was 4.83. The difference of the means was 2.44 and was used to calculate the effect size. The median moved from 2 to 5 and the mode went from 1 to 5. The range remained the same, 5, for both tests but moved from 1-6 for the pretest to 2-7 for the posttest.

Gall, Gall, and Borg (2003) stated that effect size may be used to clarify the practical significance of research results. Maxwell and Delaney (2000) opened their discussion of effect size with, “The simplest measure of the treatment effect is the difference between means” (p. 98). The “standardized difference between means” (Maxwell & Delaney, 2000, p. 99) allows the comparison of effect sizes across measures since a common scale is used. The way this is achieved is by dividing the difference of the means by the standard deviation (Maxwell & Delaney, 2000). Dr. Vicki Peyton (personal communication, May 10, 2005) verified that the average difference score would be used in the numerator and the standard deviation of the difference scores in

Table 1

Descriptive Statistics for the Results of the 1997 Study

Test	Mean	Median	Mode	Range	SD
Pretest	2.39	2	1	1-6	1.50
Posttest	4.83	5	5	2-7	1.42
Difference	2.44	3	4	0	1.29

Effect Size 1.89 (2.44/1.29 = 1.89)

the denominator in the formula to derive the effect size. The effect size of 1.89 for the sixth-grade study can be interpreted as a gain of 1.89 standard deviations based on a normal curve. The concern about applying the effect size to this study with only 18 subjects in the sample was allayed with Dr. Bruce Frey's email response (personal communication, May 2, 2005). He wrote, "Generally speaking, if you have statistical significance, you can trust it, even in the presence of a small sample size. In fact, your large effect size [1.89] was how you would have been able to achieve significance." Cohen (1992) defined a "small effect" as .20, a "medium effect" as .50, and a "large effect" as .80 (p. 157).

Another interpretation of the data used criterion-referenced scores which compared raw scores to a standard of performance. The criterion of correctly answering 4 of the first 5 problems (80% correct) was used to determine success with the identified attribute of problem-solving ability. Average students were not expected to be successful with the last two more challenging questions. Using the criterion of at least 4 correct answers on items 1-5 as the measure of success, only 3 of the 18 students, or 17%, met the criterion on the pretest. By comparison, 15 of the 18 students successfully met the criterion on the posttest. This represents 83% of the 18 students.

Using identical test items, the average difficulty level, or *p* value, for the first five problems went from .44 on the pretest to .86 on the posttest. The last two questions are not included in this comparison because they were definitively more difficult; the average *p* values for items 6 and 7 were .09 on the pretest and .17 on the posttest, respectively. The interpretation guideline offered by Sax (1997) suggested that the first five items were "hard" on the pretest and "easy" on the posttest (p. 243). Sax stated that mastery test items usually have high values of *p*, or difficulty level. This means that a high proportion of students respond correctly to the items since the intent is success for all, or mastery. This information is another indication that student achievement improved from the pretest to the posttest.

Students were instructed to show their work on both the pretest and posttest. These papers were examined for observational information about the students' algebraic problem-solving ability. On the posttest, the students were better able to organize their thoughts with a systematic means to attack the problems. Most used the problem-solving sequence devised for this class.

Validity of the test scores. A seven-question, word-problem test was compiled from Hands-On Equations Verbal Problems Sides A and B (Borenson, 1994) to measure student achievement and to use as both the pretest and the posttest. A copy of this test is in Appendix C. Problem solving was the purpose of the pretest and posttest; all seven test items were problems to solve.

The content validity of the test was intact because the seven questions directly related to student ability to solve problems that were algebraic by design. Content-related evidence is determined by content experts who can define the specific content universe or domain that the test is assumed to represent and then judge how well that content universe or domain is sampled by the test items (Gall, Gall, & Borg, 2003). A panel of three mathematics experts, each with an earned Ph.D. in mathematics, examined the test and verified its content validity. The panel of experts included Dr. Robert Fraga, Professor of Mathematics at Baker University, Dr. Jean Johnson, Professor of Mathematics at Baker University, and Dr. Mircea Martin, Assistant Professor of Mathematics at Baker University. Those verification documents are in Appendix E.

Reliability. Several factors affected the reliability of the test scores in this study. Increasing test length is one of the most effective ways to improve reliability. The seven-item test used in this study was short. While this length may have impacted reliability, the length seemed appropriate for the sixth-grade students because they needed 30-45 minutes to complete the test.

Since the same form of the test was used as both the pretest and posttest and these tests were separated by a time of 13 weeks, stability was used as one measure of reliability. In Sax's discussion of equivalence as a measure of reliability (Sax, 1997), he stated that, "Errors of measurement occur because items on two forms of a test are likely to differ. The more they differ, the greater will be the amount of *unreliability*" (p. 277).

Sax (1997) said that for non-standardized tests, measures of internal consistency, or homogeneity, are more applicable than stability, equivalence, or stability and equivalence which are most practical for standardized tests. Internal consistency was determined with the Kuder-Richardson formula 20 reliability technique. Kuder-Richardson reliabilities require items to be scored dichotomously, 0 or 1, which is the way this test was scored. Using this technique, the pretest internal reliability was

.59 and the posttest reliability was .64. Sax (1997) stated that ideally reliability should be +1.00 but decisions about groups of students are reliable with ratings “of not less than .50” (p. 293).

Design of the Study

The current study differed from the 1997 action research study; this was a qualitative study with interviews as the source of data in the study. This section gives a rationale for employing the qualitative research format in this particular study. The narrative then describes the participants in the current study and the criteria that qualified each for participation. A description of the procedures used in collecting and analyzing the data follows.

Rationale for a Qualitative Study of Hands-On Equations

Locke, Spirduso, and Silverman (2000) claimed that every graduate student desiring to employ qualitative research should answer one question honestly, “Why do I want to do a qualitative study?” (p. 114). The following discussion offers an answer to this question in relation to the current study. Maxwell (1996) claimed that the rationale for doing a particular study may be based on curiosity about a specific event. Such an event is described in this same discussion.

In the spring of 2003, two of the students who participated in the 1997 study were visiting at their lockers between classes. Their conversation centered on their perceived value of their Hands-On Equations experience. They both agreed that they had benefited from the experience and that it made subsequent, more formal algebra classes in high school easier. This casual conversation prompted this researcher to want to learn more about the perceptions of these students and others who experienced Hands-On Equations in that 1997 sixth-grade classroom. What was it about a mathematical learning experience these students had had 6 years earlier that would have prompted high school seniors to voluntarily reflect on the value of that earlier experience? Both students are strong academically and one could assume that they would have done well in mathematics classes anyway. The possible contribution to them along with a potential difference between their experiences in algebra classes and the experiences of their high school peers who did not have the Hands-On Equations exposure were initial topics of interest and prompted the current research. The interview format afforded by a qualitative study was necessary to investigate these students' perceptions.

Rubin and Rubin (1995) asserted that a “topic that is suitable for qualitative work requires in-depth understanding that is best communicated through detailed examples and rich narratives” (p. 51). Discovering the perceptions, attitudes, and feelings of high school graduates 8 years after they experienced Hands-On Equations required this in-depth understanding provided by the interview format. Rubin and Rubin added that qualitative interviewing is appropriate when one wants to learn how present situations resulted from past decisions or incidents. The design of this study matched this description.

Rippey, Geller, and King (1978) asked if one could infer learning from a student's recollection of a previous state of knowledge. They explained that it was important to have the subjects' cooperation in such a study. Their study suggested that retrospective pretesting is a viable method; students were able to recall their state of knowledge at the beginning of the course when they were interviewed at the end of the course.

The analyses of five separate studies that used retrospective ratings were described by Howard (1980). The findings suggested that retrospective self-report ratings were more reliable than traditional pretest ratings. The difference in the “then” or retrospective ratings versus the pretest ratings is called response-shift bias. The use of retrospective self-reporting eliminated this bias and produced results that were more in agreement with anecdotal, objective, and behavioral indices of the same constructs. Howard recognized that many researchers at the time were uncomfortable with retrospective measures. The reasons he gave included the logical positivism stance that all self-report instruments are suspect and response-style bias such as subject compliance, memory distortion and social desirability may be present. Howard cited studies to counter the latter concern. He also reviewed several older studies from the 1950s where retrospection had been used “quite profitably in many areas of psychology” (p. 102). He went on to say that his research “merely highlights the value of an old, undervalued research tool” (p. 103). He added that “the integration of self-report, objective, and behavioral measures has long been recognized as the most complete way to evaluate a treatment intervention” (p. 104).

More recently other researchers have utilized the retrospective approach in disparate fields (Fenwick, n.d ; Jarratt, Mack, & Watson, n.d.; Lengfelder & Heller, n.d.). Fenwick presented the Bruce Greyson Lecture at the International Association for Near-Death Studies 2004 Annual Conference. In that lecture he credited retrospective studies with the discovery of near-death experiences (NDEs). Research on this documented phenomenon relies on interviews that take place after a person has survived a near-death experience. The methods employed by Jarratt et al. included a list of interview questions and taped interviews in order to gather retrospective writing histories of college students. They shared their research results at the Writing Research in the Making Conference in Santa Barbara, California in 2005. Lengfelder and Heller employed in-depth interviews in their retrospective study. Subjects agreed to be interviewed with approximately 30-minute telephone conversations. Trained interviewers used an interview guide which contributed to semi-standardized interviews. The results of their research were included in a paper titled *German Olympiad Studies: Findings from a Retrospective Evaluation and from In-depth Interviews. Where Have All the Gifted Females Gone?* The authors presented their work at the annual meeting of the American Educational Research Association in New Orleans in 2000.

The 2000 report titled *Guiding Principles for Mathematics and Science Education Research Methods: Report of a Workshop* (Suter & Frechtling, 2000) specifically addressed the value of qualitative research in mathematics education. A group of 30 investigators, all leading researchers in their fields, were invited to attend and discuss the appropriate methods for high-quality research proposals on mathematics and science education. The segment on Education Research asserted that the growing maturity of mathematics and science education research has shifted attention from strict traditional experimental methods to alternative methods for research. More current research designs involve contributions from teachers and students. This newer research paradigm is characterized by more emphasis on "recognition of the theory-ladenness of observation and method" (p. 6) and "the implications of subjects' constructions of content and subject matter for determining meaning" (p.6). Of the 100 National Science Foundation research awards that ended between 1990 and 1998, 41 of the grants used a descriptive case study; methods of experimental design or quasi-experiment were not prevalent. This is in contrast to the 1960s when the pre-post design with randomly assigned experimental and control subjects was common. Researchers now "acknowledge that students, teachers, and education institutions are not as amenable to empirical-analytic research traditions as are the fields of psychology or agriculture, which were frequently used as models for education research" (p. 9). One distinction of qualitative research is that it may not be "geared toward the broad collection and analysis of nationally representative data, but rather toward a narrowly focused, in-depth study of interaction in a particular environment with a particular set of participants" (p. 13).

Merriam (1998) also emphasized understanding as an impetus to qualitative research. "Why?" and "What does it mean for those involved?" are questions that Merriam (1998, p. 59) claimed are associated with qualitative research and the interview process. Patton (2002) stated that the purpose of interviewing is to find out what is in someone else's mind. The intent of this research study was to learn what these young people thought and currently think about their experiences with and the effects of Hands-On Equations.

Another purpose of this study was to discover the students' perceptions of mathematics. Creswell (1994) promoted qualitative research when reality is subjective as seen by the participants in a study. Maxwell (1996) defined interpretation as the meaning given to a situation by the participants "in their own terms" (p. 32). The two high school students in casual conversation interpreted an earlier experience and attributed an outcome to it. In their words, “it [Hands-On Equations] made high school algebra easy.” As Maxwell developed his ideas about *theory* including the benefits, drawbacks, and origins, he compared theory with *description* and *interpretation*. Maxwell explained that taking either description or interpretation and constructing an explanation based on these “is to convert them into theory” (p. 32). This conversion then allows for the main purpose of theory, which Maxwell claimed is explanation. “Theory is a statement about what is going on with the phenomena that you want to understand. . .it is a *story* about what you think is happening and why” (p.32).

Taking these students’ *interpretation* a step further, one could arrive at the *theory* that Hands-On Equations with its in-depth learning of key mathematical concepts could facilitate the learning of more formal algebra. Maxwell (1996) defined theory as “a structure that is intended to represent or model something about the world” (p. 31). His definition included a broad gamut from grand theory such as postmodernism to the everyday explanations of events. His definition thus speaks to the entire range of theories rather than a particular level of complexity. The informal conclusions of the two students thus qualify as theory and, if shown to be accurate, could broaden the understanding of the contributions of Hands-On Equations. These materials purport to enhance student comprehension of the concepts of equality and variable (Borenson, 1994). What if the use of these materials could contribute to a deep understanding that impacts later learning experiences in algebra? If so, the value of these mathematical educational materials would be significant.

Participants

Two groups of students were interviewed for the current qualitative study. All of these students graduated in 2003 from Baldwin High School, Baldwin City, Kansas. Both groups of students were enrolled in the Baldwin USD 348 school district from at least the 6th grade through the 12th grade. The first group consisted of students who were in the 6th-grade classroom in which 21 lessons of Hands-On Equations were taught in January 1997. The second group was composed of students who were in Baldwin 6th-grade classrooms where Hands-On Equations were not used in January 1997.

Of the original 18 students who were members of the Hands-On Equations class and also had both a pretest and a posttest associated with those 21 lessons, 12 met the criteria of completing their education in the Baldwin schools from 6th grade through 12^{th} grade. Of these 12 students, 10 agreed to participate in this study. Since 10 Hands-On Equations students agreed to be interviewed, interview permission from 10 students who were not in the Hands-On Equations classroom was secured. Thus, interviews totaled 20, with 10 from each group. It was later determined that one of the students in the non-Hands-On Equations group moved to the district during her 8^{th}-grade year. Her interview was not included in the data since she did not meet the criterion of attending Baldwin schools from 6^{th} through 12^{th} grades. The study results reflect the data from 19 interviews.

The Hands-On Equations group included 10 students; none were minority students. Of the 7 students who received special services in the sixth grade in 1997, 5 were included in the current study; 2 were identified as gifted and 3 received assistance in the Interrelated Resource room. Of these 10 students, 1 was identified by the sixth-grade teacher as low-achieving. Of these 10 students, 6 were male and 4 were female. All graduated from high school in 2003. At the time of the interviews in 2005, 8 attended college and 2 were working.

The non-Hands-On Equations group included 9 students with usable data; none were minority students. No information was available on their inclusion in special services when they were sixth graders. Of these 9 students, 4 were male and 5 were female. All graduated from high school in 2003. At the time of the interviews in 2005, 7 attended college and 2 were working.

Data Collection Procedures

Students in the Hands-On Equations group were contacted by telephone to see if they would consent to be interviewed for this study. Permission from students in the non-Hands-On Equations group was then secured.

An attempt to approximately group students by ability and achievement levels could control for answer variations based on these two criteria. After interviews with students in the Hands-On Equations and non-Hands-On Equations groups were completed, the researcher went to the high school counselor and viewed the high school records for all students since they all granted the researcher permission to do so. The grouping of students according to ability and achievement levels took place after this viewing and recording of the GPA, ACT, and mathematics grade data. The approximate grouping of students was a desired demographic component of this research but was not crucial to the design of the study. Categories based on the ability and achievement groupings contributed to the interpretation of the data in several instances.

Interview settings and procedures. All interviews were conducted during June, July, and August of 2005. Three of the Hands-On Equations students including the researcher's daughter were interviewed by an alternative interviewer who has an earned Ph.D. in sociology and has conducted qualitative studies in the past. The researcher conducting this study interviewed the remaining 7 students in this group as well as the 9 students who were not in the Hands-On Equations class.

Students who verbally agreed to participate were interviewed in their homes (4 students), the researcher's home (9 students), by long-distance telephone using a speaker phone at the alternative interviewer's home (1 student), or at a local university library (5 students). The interviews were audio taped with the recorder started after the initial get-acquainted period and the signing of the Informed Consent Statement. Each student had the option of consenting to just the interview or to both the interview and the viewing of the subject's high school records. All 19 students consented to allow the gathering of their GPAs, mathematics grades, and ACT scores from their high school records. A copy of the Informed Consent Statement is in

Appendix F. The interviews utilized the list of protocol questions found in Appendix G. As a part of these interviews, the students were asked to solve six linear equations that were taken from the three levels of Hands-On Equations. These equations were printed on two worksheets. These worksheets are included in Appendix G.

Interview protocol questions. The interview protocol questions are listed in Appendix G. Each question was designed to elicit information about one or more of the four research questions.

Some questions applied only to students who experienced Hands-On Equations in the sixth grade. Those protocol questions were 2, 3, 12, 13, and 14; the non-Hands-On Equations students were not asked these questions. If any student answered "no" to question 4 which asked if the student took algebra in junior high, high school, or college, questions 5 and 6 were omitted for that student. All students were asked question 16 which expected the students to solve six one-variable linear equations.

The first research question dealt with the students' perceived value of the Hands-On Equations materials. Five interview questions were used to gather this information.

Protocol question 1 verified if the student was in the sixth-grade class when Hands-On Equations was taught in 1997. Questions 2 and 3 asked for any memories of and reactions to those lessons. With question 13, the student judged whether or not other sixth graders should learn Hands-On Equations. The question also probed their reasons about why they would or would not recommend these materials for other sixth graders. Question 14 asked for three recalled facts about Hands-On Equations.

Research question two asked, “Did the Hands-On Equations lessons create a student perceived difference in subsequent learning in algebra for students taught with Hands-On Equations?” Four interview questions were used to gather this information. Protocol question 4 asked if students took algebra courses in junior high school, high school or college. For those students who answered “yes,” protocol question 5 asked them to describe their experiences when they got to their first algebra class. With protocol question 6, students were asked to describe algebra as either easy or hard and to explain their answer. Question 12 asked for an opinion as to whether or not Hands-On Equations made a difference in subsequent learning of mathematics.

The third research question asked if a difference in self-efficacy exists between the two groups. Responses to three interview questions provided information on this area of the study. The answer to protocol question 6 addressed this issue by asking the students to describe algebra as either easy or hard and to explain their answer; this question was reserved for those students who had taken at least one algebra class. Students were asked to label themselves as either “good at math” or “not so good at math” in protocol question 11. Protocol question 15 asked students to recall two positive experiences in mathematics and two negative experiences in mathematics. All students were asked questions 11 and 15.

Research question four looked at group differences in three areas: (a) student attitudes toward mathematics, (b) student achievement in mathematics, and (c) student ability to solve simple one-variable linear equations. Nine interview questions were used to gather this information. For student attitudes, protocol questions 5, 6, 7, 9, 10, and 15 were utilized. Protocol question 5 asked students to describe their experiences when they got to their first algebra class. Protocol question 6 was predicted to elicit pertinent comments since it asked students to describe algebra as easy or hard. Again, questions 5 and 6 were presented only to the students who answered “yes” to question 4. Protocol question 7 asked students to share their recalled grades in their mathematics classes. The purpose of this question was to prompt any additional memories of algebra and/or mathematics, and student reactions to the grades received. Protocol question 9 asked students to rank order 10 subject areas: art, health, language arts (English), math, music, physical education (PE), reading, science, social studies, and writing. Question 10 probed for the reasons for the mathematics ranking. Protocol question 15 asked for two positive and two negative experiences in mathematics with the intent of prompting responses that might answer this first part of research question four. All students were asked questions 7, 9, 10, and 15.

For student achievement, protocol questions 4, 7, and 8 contributed information. Protocol question 4 asked what courses the student took in junior high school, high school or college. Protocol questions 7 and 8 asked students to share their recalled grades in their mathematics classes and their ACT scores. All students granted permission to the researcher to view their high school transcripts and record their grades and ACT results which also provided data pertinent to this research question. All students were asked questions 4, 7, and 8.

Protocol question 16 was the only question that dealt with a student's ability to solve simple linear equations. Students were asked to solve six one-variable linear equations. All students were asked to respond to this question.

Data Analysis

The audiotapes of the interviews were transcribed for the 19 students. These transcriptions, the six equations worksheets, and the students' high school records constituted the data. Wolcott (1994) wrote an entire book about transforming qualitative data, *Transforming Qualitative Data: Description, Analysis, and Interpretation*. His emphasis on transformation of "unruly experience" (p. 10) supported his belief in the importance of dealing with the dilemma of what to do with the data in qualitative studies. One of the ways he suggested to organize and present description is to follow an analytical framework. The research questions used in this study provided such a framework. When discussing analysis, Wolcott added the idea of fleshing out whatever analytical framework guided that data collection. He also said to "identify patterned regularities in the data" as a means to discern "what-goes-with-what" (p. 33); this categorization of the data contributes to understanding beyond the limits of the small samples in qualitative research projects.

Using Wolcott's (1994) framework concept and his idea of categorization for meaning finding, the data was analyzed by color coding the parts of the transcripts that correspond to the various research questions. Another of Wolcott's suggestions was to display the findings in tables, charts, diagrams, and figures. Data were organized in tabular form and included in Chapter 4. These tables address student memory for Hands-On Equations, student GPAs, ACT mathematics scores, equation solving scores, and mathematics rankings, and were used to investigate any patterns among students with varying achievement levels.

Self-confidence data were identified and organized using the categories devised by Esty and Teppo (1994). They organized their interview data according to mathematical self-confidence in students. The self-confidence characteristic was labeled as either "non-confident" or "confident" with two non-hierarchical, descriptive sub-categories under "non-confident" and three non-hierarchical, descriptive sub-categories under "confident." Adaptations of their two classifications in the area of self-confidence were also used in the present study:

Non-Confident

1. Inaccessibility of mathematics: Students indicated that forces beyond their control impeded their ability to do mathematics. They may have perceived mathematical activities and learning to be meaningless and difficult to understand.

2. Lack of accomplishment: Students indicated that they perceived few positive results despite working hard. They also indicated they lacked specific skills and knowledge.

Confident

3. Accomplishment: Students indicated that they could perform specific mathematical activities and/or commented that, by working hard, they were able to succeed.

4. Confidence: Students expressed feelings of confidence over their performance in mathematics and commented on how motivated they were to do mathematics.

5. Understanding: Students commented on their ability to understand mathematics.

As other categories emerged from the collected data, further adaptations were made. The work of Esty and Teppo served as organizational guidelines but the final product was based on the interview data from this study.

Attitude results were based on the student responses during the interviews. The responses included the rankings assigned to mathematics by the students as well as student comments about liking or disliking mathematics.

The data on GPAs, mathematics grades, and ACT scores were reviewed (a) for any discrepancies between the two groups of students and (b) to see if there were any relationships between student interview responses and the data from the ACT and GPA scores and mathematics grades.

Pilot Study

A pilot interview was conducted with one of the six students who had both a pretest and posttest score in 1997 but who did not attend Baldwin schools from 6^{th} through 12^{th} grades. This student is representative of the 1997 Hands-On Equations group since he attended another school for a brief time before returning to Baldwin and finishing his high school education. The student readily agreed to the interview which was held in the researcher's home on March 30, 2005.

Interview Results

This subject was very positive about the value of the Hands-On Equations manipulative materials. He remembered "thinking that it was fun." He described himself as "fairly good at math" and thought that Hands-On Equations "helped in the beginning" when he started to do "higher math" which is what he called his eighth-grade algebra class. He was asked, "Do you think other sixth graders should learn Hands-On Equations?" His answer was “yes.” The reason he gave was Hands-On Equations made it “funner and easier to learn the math.”

Bruner observed (1966) that in going from the concrete to the abstract, the learner retains and draws upon the “store of concrete images that served to exemplify the abstractions” (p.65). Interestingly, the pilot study subject described the Hands-On Equations materials in a similar manner; he seemed to exemplify Bruner’s research conclusion. When the subject was asked why he thought that Hands-On Equations helped when he got to algebra classes, he responded, “It was just a lot easier to think of like, the little pieces.. . .with the simple beginning problems, it was easy to imagine that and to think, well, it gave you something to visualize. And it wasn’t just marks on a piece of paper. You could actually think about it in your head, and picture it and it made it a little more real life.” The transcript of this interview can be found in Appendix H.

The pilot subject worked the equations at the end of the interview session. A copy of these equations is found in Appendix G. He quickly and accurately worked all six of the equations. This task took him approximately two minutes to complete.

When presented with the student's signed Informed Consent Statement, the high school principal approved the request and referred this researcher to the high school counselor for access to the student's records. This student's high school scores and mathematics grades were strong.

Resulting Modifications

Several modifications to procedures were made as a result of this pilot. It is important to explain the Informed Consent Statement before having the student read and sign it. The formality of the form may be intimidating to someone who has not seen such a document before. Also, the request to view confidential data such as test scores and grades may shock unless their purpose and intent are explained prior to reading. Since the request to view grades and test scores was not explained ahead of time, the subject expressed surprise when he read that portion of the Informed Consent Statement. He did agree and signed the form.

A change was made to the Informed Consent Statement based on the pilot interview. The subjects in the study had the option of consenting to just the interview or to both the interview and the viewing of the subject's high school records.

The Informed Consent Statement in Appendix F incorporated this revision.

The list of protocol questions was rearranged as a result of the pilot interview. The original placement of the six equations to solve was number 11 of 15 items. During the pilot, this timing of the equation solving seemed awkward and was switched to the end of the interview. This worked well and was continued with the actual research interviews. The protocol questions page was revised to reflect this change and can be found in Appendix G along with worksheets that have the six equations on them.

CHAPTER 4

ANALYSIS OF THE DATA

Introduction

This qualitative research study was designed to examine the perceptions of high school graduates who experienced the mathematical materials from Hands-On Equations when the students were in the sixth grade. The investigation also included the perceptions of students who did not experience Hands-On Equations during their sixth-grade year. The study was conducted 8 years after the students were in the sixth grade.

The results of the analysis of the data are divided into five sections. The first three of these five sections deal only with the data associated with the students who experienced Hands-On Equations when they were in the sixth grade. The first section describes student memory of Hands-On Equations. The next four sections correspond to the four research questions. The second section reports the perceived value of Hands-On Equations. The third section deals with the perceived difference that Hands-On Equations made for students in subsequent algebra courses. The fourth section examines mathematical self-efficacy for all students in the study. The fifth section analyzes differences between the two student groups related to student attitudes toward mathematics, student achievement in mathematics, and student ability to solve simple linear equations.

Four research questions guided this study.

1. For the students who experienced Hands-On Equations, what is the perceived value of these materials?
2. Did the Hands-On Equations lessons create student perceived differences in subsequent learning in algebra classes for students taught with Hands-On Equations?
3. Is there a difference in present mathematics self-efficacy between students taught with Hands-On Equations and those who did not experience these teaching materials?
4. Are there other differences related to (a) student attitudes toward mathematics, (b) student achievement in mathematics, and (c) student ability to solve simple linear equations between students taught with Hands-On Equations and students who were not?

Student Memory of Hands-On Equations

The 10 students who experienced Hands-On Equations were first asked if they remembered doing Hands-On Equations when they were in the sixth grade; 9 answered affirmatively. This was a crucial question because there would have been no study if the participants had remembered nothing about the subject of the study. Table 2 summarizes the students' memory of Hands-On Equations. The Hands-On Equations students are labeled from 1-10 on Table 2 and subsequent tables; the non-Hands-On Equations students are labeled from 11-19 on subsequent tables. On all student data tables, students were rank-ordered within their group by their high school overall grade point averages to discern any possible patterns between higher- and lower-achieving students and to supply contextual information about the students. Further student descriptive information is located in Appendix I.

Included in Table 2 is whether or not a particular student could recall Hands-On Equations without the prompt of the student materials (the laminated mat and the manipulative pieces). The "Yes/No" delineation indicates that the student recalled something without the physical prompts but remembered additional information when shown the student materials. Four

Table 2

Amount and Type of Student Memory of Hands-On Equations

Student	GPA	Recall Without Prompt	Amount of Recall	Reaction Memory Present	Type of Reaction Memory
1	4.00	Yes	Much	Yes	P-A
2	3.84	Yes	Much	Yes	P-A
3	3.43	Yes	Much	Yes	P-A
4	3.06	No	None	No	
5	2.97	No	Limited	Yes	P-E
6	2.94	No	Limited	Yes	P-E
7	2.89	Yes	Much	Yes	P-A
8	2.70	Yes/No[a]	Some	Yes	P-A
9	2.64	Yes/No[a]	Limited	Yes	P-A
10	2.13	Yes/No[a]	Some	Yes	P-E

Note. Four categories were identified for the reaction memories: P-A = Positive Academic; N-A = Negative Academic; P-E = Positive Emotional; N-E = Negative Emotional.

[a]Indicates that the student recalled something without the prompt of the student materials but remembered additional information with the presentation of the student materials.

students remembered easily without the prompt (“Yes” on Table 2), three remembered something about Hands-On Equations but recalled more when presented with the physical prompts of the Hands-On Equations materials (“Yes/No”), two more needed the physical prompts in order to revive the experience (“No”), and the third “No” student did not recall Hands-On Equations even when presented with the prompts. In summary, 9 of the 10 students remembered Hands-On Equations to some degree with or without a physical prompt.

The amount of recall of Hands-On Equations is also included in Table 2. The recall data were gathered from the transcribed interviews which were structured by the protocol questions. These questions prompted student answers. Descriptors—“Much,” “Some,” “Limited,” and “None”—were used to categorize the overall amount of recall. "Much" described the responses from four students who provided multiple sentences per answer about remembering Hands-On Equations in numerous question/answer exchanges. Two students had "Some" recall which was not as abundant as the "Much" category. Three students recalled very little; they either responded with one or two sentences or, at times, said that they did not remember when asked questions. Their responses were labeled "Limited." And one student used the wording “not very much” when asked what he recalled about Hands-On Equations which might imply that he recalled something. This was not the case; he could relay no memories of experiencing these materials. It was determined that his “not very much” was his way of avoiding saying “no” but meaning “no.” His recall was labeled “None.”

When the Hands-On Equations students were asked if they remembered any reactions to experiencing those materials when they were in the sixth grade, four categories emerged from the responses of the nine students who had recall. These four categories are denoted in Table 2. First, whether or not a student had a reaction memory was noted with a “Yes” or “No.” Nine students remembered their reactions at the time to the sixth-grade Hands-On Equations lessons. Consistently, the student with no general recall of Hands-On Equations had no reaction memories as well. Reaction memories were labeled as either academic (A) or emotional (E); comments that predominantly described what the student learned were termed “academic” and comments that predominantly displayed feelings toward the Hands-On Equations experience were labeled “emotional.” These academic and emotional reactions were further categorized as either positive (P) or negative (N). Thus the four categories discerned from the data that described the reaction memories were positive academic, negative academic, positive emotional, and negative emotional.

As stated, nine students remembered something about Hands-On Equations but the amount of recall varied. The four students who had "much" recall shared vivid and detailed memories. When asked to recall any reactions to Hands-On Equations, they all gave answers related to academics rather than emotion. Three of these four students had the highest grade point averages (4.00, 3.84, and 3.43) and the highest mathematics ACT scores (28, 32, and 22, respectively). Student 1, as labeled on Table 2 and in subsequent tables, made the following comments.

> We all had little sheets of paper that were supposed to look like the balance scale. And on our desks and up at the front of the class, we had like little manipulatives, and they were supposed to represent our unknowns, our variables.

Student 2 offered, “You had like the little figures that represented Xs and Ys and everything and I remember it was just real visual because you could see, you know, because you had to keep it in balance like a scale." Student 3 gave his remembrances.

> I remember balances . . . it [was] all about the equal sign. It was all about moving things from one side to the other and what happens and what you have to do and apparently taught me the initial rules of algebra.

Student 7 offered, "I remember having this like a teeter totter scale and we were learning that, you know, how to balance it out."

Two students had what was termed "some" memory of Hands-On Equations; one had an emotional response while the other had an academic answer when asked to recall any reactions to Hands-On Equations. Student 10 answered, "More fun. More interesting. . .than just somebody speaking at you." Student 8's academic response had emotional aspects, "I remember liking the fact that it was a scale and it kind of showed you how to balance the sides. That was pretty cool."

Three students could remember very few specifics; they had a "limited" amount of recall. Of these three students, Students 5 and 6 recalled strong, positive emotional responses to Hands-On Equations. They both volunteered identical responses, "I liked it." Student 9 said that it "helped me to put things together. That's a positive thing." This reaction was labeled an academic memory rather than an emotional memory in Table 2. Of the nine students who recalled either an academic or emotional response, all nine expressed positive reactions to Hands-On Equations in the sixth grade.

Later in the interviews, students were again focused on recall when they were asked to provide three facts about Hands-On Equations. The response from Student 1 included much detail.

> One, is what you do to one side, you have to do to the other. Another is when you have one little piece, one little—whatever it is, your little thing that you're taking off that represents your variables—when you have that alone, that equals whatever is left on the other side, the—whatever like your numbers represent. I can't remember what the numbers were. But once you have that one side alone, that's. . .you always want to get it by itself. So you're always constantly like working to like take things away or add things to the other side. You're always constantly working so you just have one thing left on one side. Then, let's see, the third fact, variables can be called anything. Like they can be *n*, which is what you

> see a lot, but you can have them be any letter of the alphabet or anything that you want. It's just an unknown. It's a variable. So it doesn't have to be *n*.

Student 2 volunteered the following.

> I remember scales. The scales and that gave you the basic idea of what you do to one side you have to do to the other side. I remember the little black and white pieces putting on the scale so we could physically take them away and you know, so it wasn't just numbers, you could physically see why that was the same.

The recall from Student 3 dealt with the concept of equality.

> I can't remember this directly but I'm sure that both sides of the equation have to be equal. Kind of the gimmie. . . Three facts. I can definitely think of three facts for algebra but I don't think they should count because I don't remember them being given in the class. And I don't know why I can't remember that, I just, I think, okay, well, *x* represents a number where the—what am I trying to—I'm thinking too fast here. Yeah, *x* represents something. I'm thinking of, if you—well, for instance, I'm thinking like *x* plus 7 equals 9. If you subtract 7 on this side of the equation you would have to subtract it on

> the other side. I'm thinking of addition but that's kind of almost the same as everything has to be equal, but it's kind of the subcategory that's pretty important.

Student 7 added the following.

> We had our equipment on our desks. And I remember having to see what you had on the board and then physically hands-on do it on our desk, put so many on one side and so many on the other side and see how it equaled out.

Student 8 could only remember one fact.

> The first fact was I remember you have to balance the scale, keep everything balanced, that's how equations worked. The two facts, there was—I don't remember. You know, I don't remember other facts. I can give you one fact. The balancing the equation. That's all I remember.

The remaining five students could remember little to no further factual information at this late point in the interview; this question about recalling three facts was number 14 out of 16 protocol questions. Student 5 said, "As far as factual recollection, it's very minimal. It's—I can't remember. I'm sorry, I can't remember much in this aspect." Student 10 responded to this question by saying, “I don’t remember really.” Student 6 gave “No” for an answer. Student 9 answered, “It helps you with your multiplying, dividing, and adding and subtracting.” After a six second pause, she asked, “Three facts?” and thought for eight seconds more before saying, “I can’t think of anything.” When asked if he could recall three facts about Hands-On Equations, Student 4 responded with, “I don’t think I could do that.”

Perceived Value of Hands-On Equations

Student perceptions about the value of these teaching materials were determined when students were asked if they thought other sixth graders should learn Hands-On Equations, and why or why not. Students offered value statements in response to other protocol questions, as well. Data related to the first research question which dealt with the perceived value of Hands-On Equations are summarized in Table 3.

One of the reasons that students recommended Hands-On Equations was because they felt that it had benefited them and would do the same for other sixth graders. Student 1 answered, “I definitely do.” She explained her reasoning.

> I think you can probably never start too early, as far as at least learning the basics of it—of math. I mean, in all

Table 3

Student Responses to Whether or Not Other Sixth Grade Students Should Experience the Hands-On Equations Materials

Student	GPA	Response
1	4.00	Yes
2	3.84	Yes
3	3.43	Yes
4	3.06	Yes
5	2.97	Yes
6	2.94	Yes
7	2.89	Yes
8	2.70	Yes
9	2.64	Yes
10	2.13	Yes

> subjects, but since we're talking about math, you can never start too early. And it's a way of teaching that even though it's algebra and it's supposed to be difficult, it's a way that kids can still learn. Like it's—the concepts are simple enough—that they can still grasp it. And the fact that it is hands-on. I mean, it probably just like keeps them interested as opposed to just learning something, you know, by memorization or something else. I just think it does have its advantages. When you get older, it's nice to have that background.

Student 2 employed the same word—definitely. He kept gesturing with his hands to show a balance scale while he gave his reasons for his response.

> Definitely. I mean, we all—we had some fairly good math students in our classes so and I mean it's definitely helped me a lot, so I really—I mean, we picked it up that easily. You know, like it was just—it became—I mean, at the time we were using Algebra 1, it was almost second nature. I could go that quick. . .It really did help me.

Student 3 also chose the same word, "Definitely helped me. Definitely." He added, "It was good because I think it would kind of raise interest." He also described the value of the experience as a "good introduction" or preparation and noted that it helped later on in an actual algebra class.

> It's just kind of the foundation that I built everything else on. Definitely a good thing, yeah. . .It was a good introduction. It was kind of an introduction that wasn't really—it wasn't so much focused on results, it was exposure kind of. It seems like if you didn't get something or something was kind of difficult, . . . you still have a while before you even start this [algebra] class.

Students 1 and 3 termed the experience valuable because it raised student interest. Student 10 also mentioned student interest as a value of Hands-On Equations. He gave an emphatic, "Oh, yeah!" as his answer to the value question. When asked why he gave such a strong response, he explained, "Oh, just because I wasn't very interested in math, but I was more interested doing that. It would keep their attention a little better."

Student 5 emphasized the value of Hands-On Equations because of student learning styles.

> I think they should. I mean, they should at least be exposed to that because, like I said, different learning styles. And you just—you just never know with these

> people—sixth graders. They have different learning styles and different levels of attention and so I'm thinking that maybe for people that maybe just don't get the—like the book aspect or the writing-down aspect. This might be a little bit more easy or more useful for them because it's more practical, you know.

Student 8 focused on visual learning in his answer to the question. "I think they should. I think it's because it's a visual learning is how [it's] learned and it kind of makes them understand that you've got to balance the sides out because it's a scale."

Four students, Students 6, 7, 9, and 4, mentioned the importance of the hands-on approach. Student 6 had earlier referred to herself as a "hands-on person." When asked if she thought that other sixth graders should learn Hands-On Equations, she said, "I think so. Just because the same reason that I had. If they are more hands-on, then it would help them a lot more." Student 7, who was labeled learning disabled while in school, also emphasized the hands-on approach and different learning styles.

> Uh-huh. I think hands-on does—every kid does learn different. And I've learned that because I'm a different

> learner from all the other kids in my grade, and I've had to learn that by having help all through junior high and high school from the resource room and them showing me different learning methods. And they had to get to know me and how I learned. And hands-on is one of the things . . .hands-on really helps me to learn. It gave the kids a chance to see if hands-on helps them and they can figure out how they learn best.

In response to the question about whether other sixth graders should learn Hands-On Equations, Student 9 said, "Yeah" and explained, "It will get them started" and help them because when students get to junior high school, "they start getting the actual math" without the help of hands-on manipulatives. She went on to say, "When they can use their hands, and use the blocks and the colors, they just. . .if they do it with their hands maybe they might remember it for the future." When Student 4 was asked if he thought that other sixth graders should learn Hands-On Equations, he responded, "I would have to say 'yeah,' because if they are a hands-on kind of learner then that would help them learn it more. It's easier to learn something hands-on sometimes than it is out of a book." Since Student 4 is the one whose recall for Hands-On Equations was deemed "none," his answer must be interpreted as generally supporting the value of hands-on learning and indirectly supporting Hands-On Equations since these materials involve hands-on activities.

During the interviews at various times during the discussion, five students specifically labeled themselves as "hands-on" learners. Student 7: "I'm more of a hands-on learner. Hands-on really helps me to learn." Student 4: "I learn more hands-on." Student 5: "I'm more of a hands-on visual guy." Student 6: "Because I'm more of a hands-on person." Later, she repeated herself, "Since I am more of a hands-on person." Student 9: "I'm hands-on." Often, these students offered this self-evaluation with a tone of voice that suggested that they were different from other learners since they preferred the hands-on approach. In this study, 50% of the Hands-On Equations students identified themselves as "hands-on" learners.

In summary, the perceptions of the value of these materials were gathered when the students were asked if other sixth graders should learn Hands-On Equations and from various responses to other protocol questions. Although their reasons varied, all 10 students, or 100%, recommended Hands-On Equations for other sixth-grade students. Again, the fact that Student 4 could not specifically recall Hands-On Equations leads one to interpret his positive response to this question as only an indirect approval of Hands-On Equations. The reasons for valuing the Hands-On Equations materials included the access to foundational algebraic knowledge that would help students when they got to their first algebra class, alignment with visual or hands-on learning styles, and the promotion of student interest in mathematics.

Perceived Differences in Subsequent Learning of Algebra

The second research question asked if the Hands-On Equations lessons created student perceived differences in subsequent learning in algebra classes. Protocol question 12 specifically asked if the Hands-On Equations experience made a difference in student learning later on in high school or college mathematics classes; the protocol question did not limit student responses to algebra classes. Protocol questions 4, 5, and 6 were also related to the second research question.

Of the 10 Hands-On Equations students, 9 answered affirmatively to Protocol Question 12. Only 9 of these 10 students took an algebra class in junior high school, high school, or college. Of the 9 students who took an algebra class, 8 perceived a difference with 5 specifically describing a difference in algebra classes. The Table 4 column labeled “Perceived a Difference” includes this data.

Protocol question 6 asked students to describe algebra as easy or hard. This data directly relates to the second research question and is also included in Table 4. Seven of the nine Hands-On Equations students who took an algebra class deemed algebra to be “easy.” Student 6 called algebra “hard” and Student 9 described algebra as both “not easy” and “not so hard.” Her answer was unclear and was categorized as “neither” on Table 4.

Eight students claimed that Hands-On Equations made a difference for them. In response to protocol question 12, Student 2 commented on the mental image that remained with him in subsequent algebra classes.

> I remember it like later in high school, taking algebra. Like thinking . . .I’m doing this so I better do it to this side, too, you know, it just really helped you get that mental image, the scale was always kind of up there.
>
> *Interviewer: The scale was up there in your mind?*
>
> Uh-huh, you could almost visually see a scale on what you were doing. I used it all the way through on algebra. It seemed like at the time [sixth grade] just like we were just

Table 4

Student Responses About Perceived Differences in Subsequent Learning in Mathematics and/or Algebra Classes and Whether or not Algebra Was Judged to Be Hard or Easy

Student	GPA	Perceived a Difference	Labeled Algebra as Hard or Easy
1	4.00	Yes[a]	Easy
2	3.84	Yes[a]	Easy
3	3.43	Yes[a]	Easy
4	3.06	Yes	Easy
5	2.97	No	Easy
6	2.94	Yes	Hard
7	2.89	Yes	Easy
8	2.70	Yes[a]	Easy
9	2.64	Yes[a]	Neither
10	2.13	Yes	NA

Note. NA = Not applicable because the student did not take any algebra classes.
[a]Student specifically volunteered that Hands-On Equations made a difference in subsequent algebra classes.

> learning math. It didn't seem like too complicated. It wasn't until later I realized what we did. Having that like a firm grasp of basic algebra, that just carries over because you don't struggle with the little things. You can just focus on what was new.

Student 1 offered her perspective.

> Algebra always came easy to me. It just completely makes sense to me. It just—for some reason, it all—it all is very clear. Like if you have an unknown, it's very easy to solve that equation. It just comes to me, I guess. And I'm sure just because I've been doing it for such a long time that a lot of it really is just second nature. I don't have to think, you know, about the rules anymore. I just do it.

When specifically asked if Hands-On Equations made a difference for her, she responded affirmatively.

> I do think so. I think it's not like a conscious awareness, but because I know what that taught me, I can't imagine what other people think about algebra. Just because like the whole idea of having a balance. When I got into more difficult—well, at the time what was difficult math, like when we started foiling and everything else, and equations got really long and messy—it was still really easy to just

> check and make sure everything was okay. I'm just assuming that the Hands-On Equations is what did that for me, because it got the idea in my head.

Student 3's comments centered around his comfort level.

> . . .the sixth-grade hands-on made me feel definitely comfortable. I always felt real comfortable in junior high with math. I can remember in that class [first algebra class] just getting something right, you know, and a lot of people not really understanding.

When specifically asked if he thought that Hands-On Equations made a difference for him, he answered, "Oh, definitely. Yeah, I do." He continued to explain why.

> Equal sign. I remember that was like—it seems to me that was like the main focus. All the physical activity of picking up something and having to think about moving it to another side and knowing why that happens. That's what helped. That's why it was useful.

"Yeah, it definitely helped," were Student 7's words in response to the same question. One of the benefits on which she focused had to do with negative numbers.

> But it really helps you to understand negative numbers. That's a big thing is having us switch over to learn how

> you can get negative numbers. It helped later on. It's confusing to know that you're going to get—you know, your whole life you've just done add, you know—and you get a positive number. And then to think you're going to get less of—it's not even there, you're minus something, nothing's even there, is kind of confusing.

Student 8 had not thought about the value of Hands-On Equations until asked if the Hands-On Equations experience made a difference for him in learning mathematics later on in high school. He responded with, "Now that I think about it, yeah. I think it really did. Because that's why algebra always came easy for me. Because I started learning it at such a young age." He described his experience in his first algebra class, "It was actually one of the classes I didn't have too much trouble in." When asked why, he attributed his success to Hands-On Equations.

> I think it was because we had all the extensive practice in junior high. I mean, I went through—sixth grade we went through the whole thing of it and I don't think any of the other sixth-grade classes did it, did they? And then when I came to pre-algebra, it was easy and then after that it just got easier.

When asked if Hands-On Equations made a difference in subsequent mathematics classes, Student 6 said,

> Yeah, since I am more of a hands-on person I think it did.
>
> *Interviewer: Why do you think it made a difference? Did you understand it when we did this [in the sixth grade]?*
>
> Yeah. I remember I did understand it.
>
> *Interviewer: You remember that you did?*
>
> Uh-huh. I thought it was easier to do it that way.

Student 9 described her experiences when she got to her first algebra class, "I thought it was going to be very hard and it turned out to not be." Even Student 4 attributed a benefit to the Hands-On Equations that he could not specifically remember. Knowing that he experienced this hands-on approach in the sixth grade, he answered protocol question 12 by concluding, "I would probably have to say yeah, because I learn more hands-on."

Student 5 was the one student who did not think that Hands-On Equations made a difference for him. "I can't really say it's made much of a difference because most of the stuff I learned in high school and college was a little bit more, you know, advanced."

In summary, protocol question 12 asked, "Do you think that the Hands-On Equations experience made a difference in your learning math later on in high school or college? If so, how? If not, why not?" As phrased, the protocol question did not limit the students' responses to algebra classes only. Nine of the 10 Hands-On Equations students took an algebra class at some point in their academic careers. Since the research question dealt only with algebra, the responses of these 9 students were the only ones used to answer the research question. Of these 9 students, 8 of them, or 89%, expressed the opinion that Hands-On Equations made a positive difference for them in subsequent learning in mathematics classes. Of these 8 students who felt that Hands-On Equations made a difference, 5 of them specifically credited Hands-On Equations with making a difference in subsequent algebra classes.

Self-Efficacy

The third research question asked if a difference exists in present mathematics self-efficacy between students taught with Hands-On Equations and those who did not experience these teaching materials. Protocol questions 6, 11, and 15 directly related to the third research question. Protocol question 6 asked students to describe algebra as easy or hard and to explain their answers. Protocol question 11 asked students if they thought of themselves as either "good at math" or "not so good." Protocol question 15 requested that the students tell about two positive and two negative experiences in mathematics. In addition to the responses to protocol questions 6, 11, and 15, comments related to self-efficacy that occurred throughout the interviews were included in the data on this topic.

Responses Related to Confidence and Non-Confidence

Self-efficacy data were analyzed according to the self-confidence guidelines developed by Esty and Teppo (1994). The self-confidence characteristic was labeled as either "non-confident" or "confident" with two non-hierarchical, descriptive sub-categories under "non-confident" and three non-hierarchical, descriptive sub-categories under "confident." Adaptations of their two classifications in the area of self-confidence were used in the present study:

Non-Confident

1. Inaccessibility of mathematics: Students indicated that forces beyond their control impeded their ability to do mathematics. They may have perceived mathematical activities and learning to be meaningless and difficult to understand.

2. Lack of accomplishment: Students indicated that they perceived few positive results despite working hard. They also indicated they lacked specific skills and knowledge.

Confident

3. Accomplishment: Students indicated that they could perform specific mathematical activities and/or commented that, by working hard, they were able to succeed.

4. Confidence: Students expressed feelings of confidence over their performance in mathematics and commented on how motivated they were to do mathematics.

5. Understanding: Students commented on their ability to understand mathematics.

The interview transcripts were carefully read for comments that indicated mathematical self-efficacy as determined by either student confidence or non-confidence with mathematics or algebra. Each individual student interview transcript was read and comments throughout the entire interview were coded with a 1, 2, 3, 4, or 5 which corresponded to the adapted categories of Esty and Teppo. The data for students who experienced Hands-On Equations are in Table 5; the data for students who did not experience Hands-On Equations are found in Table 6. Students were rank-ordered within each of the two groups by their high school overall grade point averages. A student who had at least one comment in a particular category (1, 2, 3, 4, or 5) received an “X” in that number column in the appropriate table for his or her group. In this study, students with at least one X in the 1 or 2 categories were labeled as non-confident. Students with at

Table 5

Presence of Categorized Student Comments Related to Mathematical Confidence and Non-Confidence for Hands-On Equations Students

		Non-Confident		Confident		
Student	GPA	1	2	3	4	5
1	4.00			X	X	X
2	3.84			X	X	X
3	3.43			X	X	X
4	3.06				X	X
5	2.97	X	X			
6	2.94	X	X			
7	2.89				X	X
8	2.70			X	X	X
9	2.64					X
10	2.13	X				
Totals		3	2	4	6	7

Note. Non-Confident 1 = Inaccessibility of mathematics; Non-Confident 2 = Lack of accomplishment; Confident 3 = Accomplishment; Confident 4 = Confidence; Confident 5 = Understanding.

Table 6

Presence of Categorized Student Comments Related to Mathematical Confidence and Non-Confidence for Non-Hands-On Equations Students

		Non-Confident		Confident		
Student	GPA	1	2	3	4	5
11	4.00			X	X	X
12	3.88			X	X	X
13	3.49			X	X	X
14	3.44			X		
15	3.28			X	X	X
16	2.93				X	X
17	2.75	X	X	X	X	X
18	2.58	X	X			
19	2.14	X	X	X	X	
Totals		3	3	7	7	6

Note. Non-Confident 1 = Inaccessibility of mathematics; Non-Confident 2 = Lack of accomplishment; Confident 3 = Accomplishment; Confident 4 = Confidence; Confident 5 = Understanding.

least two Xs in the 3, 4, or 5 categories were deemed confident.

Comments from Hands-On Equations students related to confidence and non-confidence. Students 1, 2, 3, 4, 7, and 8 of the Hands-On Equations group gave comments that indicated that they felt confident with mathematics. Each of these six students made comments that fell into two or more of the categories labeled 3-Accomplishment, 4-Confident, and 5-Understanding.

Students 1, 2, 3, and 8 relayed their sense of accomplishment in mathematics (Category 3). Student 1 asserted the following.

> The reason why I had the most success at math was because I always did my homework. So I learned it. I mean, I really learned it as opposed to just kind of, you know, getting by until a test and forgetting.

Students 2 and 3 mentioned high mathematics grades and doing well in state mathematics contests. Student 8 qualified his positive responses as applying only to algebra. When asked to recall two positive and two negative experiences with mathematics, he shared descriptions of success with algebra but dismal experiences with geometry and trigonometry.

> We'll start with the negative because those are the easiest. Taking trigonometry my senior year, I had a tutor and it

> just didn't help and I ended up getting a pretty bad grade. And I think my other experience was in geometry. I didn't get a horrible grade but the whole semester I was lost and I could never get back. The two positive experiences were I got an A in my math class in freshman—or sophomore—year when I took algebra. And then one of the toughest classes they said [was] at Baldwin High School was Algebra 2 and I didn't do very bad in that one, also.

As opposed to Student 8, Students 1, 2, and 3 felt accomplished in mathematics in general.

All six students who were generally considered to be confident, Students 1, 2, 3, 4, 7, and 8, expressed feelings of confidence (Category 4). Student 1: "Algebra always came easy to me. I think I'm good at math." Student 2: "Algebra is very easy. [I'm] fairly good at math. . .As far as what I have learned, I feel comfortable". Student 3 explained his comfort level.

> The sixth-grade Hands-On made me feel definitely comfortable. I think the biggest thing for a kid that age is to figure out why the *x* is there, you know. But it really wasn't a—it wasn't a shock and I felt—I always felt real comfortable in junior high with math. Personally, algebra is easy.

Student 4 shared his thoughts.

> I think of myself as being pretty good [at math]. It's easy if you do it often and if you remember what it is they want you to do with the numbers and everything. [I'm] pretty good with math compared with some other stuff. Algebra is okay and I'm okay at algebra.

Student 7: "Math is an easier subject for me. [I'm] good at math. . . High school math was easy for me." Student 8: [Algebra is] easy, I think.

Understanding (Category 5) was either directly stated or implied in statements by Students 1, 2, 3, 4, 7, 8, and 9. Student 1: "It just completely makes sense to me. It all is very clear." Student 2: "[It] was all just really basic to me. . .I remember a couple of other people in my class understood it real well, too." Student 3: "Personally algebra is easy. . .I've always enjoyed math and. . .the sixth-grade Hands-On made me feel definitely comfortable." Student 4: "It's (mathematics) just one of the things I understood more and that was more easy." Student 7: "I could really understand. . .Like when you have an equal sign and you have to balance out both sides, make both sides equal to one another, I can really do that." Student 8: "I understand algebra and other parts of math I do not understand. . .Algebra always came easy to me." When asked if she understood algebra in her first class, Student 9 explained that she understood it within a few weeks of the beginning of the class.

Interviewer: Did you understand algebra in that first class?

Not at first and then I did.

Interviewer: How long did it take to figure it out?

Probably a couple of weeks.

Students 5, 6, and 10 expressed feelings that were termed non-confident. Mathematics was deemed inaccessible (Category 1) to these students. Student 5: "I know I'm not the best math student ever to understand. . .Math is not really my subject." Student 6: "It (first algebra class) was kind of hard to me. I'm not really a math person. . .I never have really been good at math." Student 10: "[I'm] not so good. Math doesn't come to me real quick."

Students 5 and 6 also commented on a lack of accomplishment (Category 2). Student 5 said, "I wasn't getting it at all. . .I'm a visual and somewhat of a slow learner when it comes to the math field, I would say." Student 6 said, "It just doesn't come easy to me and I don't really remember a lot of things about it."

Student 9 appeared to be neutral in regards to confidence. She expressed neither confidence nor a lack of confidence but she did admit to a degree of understanding of algebra; this is noted on Table 5.

With three categories in the “confident” area and ten students, there were 30 opportunities where self-confidence could have been denoted on Table 5. The data showed 17 marked affirmative categorizations or 57% of the possible opportunities to express confidence. Two categories in the “non-confident” section of Table 5 and ten students meant that there were 20 slots for recording non-confidence. Of these 20 possibilities, 5 were recorded which equaled 25% of the possible opportunities to express non-confidence for the Hands-On Equations group.

Comments from non-Hands-On Equations students related to confidence and non-confidence. When considering the data from the non-Hands-On Equations students, it was found that Students 17 and 19 described mediocre achievement in high school but had since discovered some success with mathematical tasks in either junior college, in one case, or in a job-training program, in the other. These two students were labeled on Table 6 as both confident and non-confident since they expressed both characteristics. Student 17 described her earlier and later experiences.

> [In] junior high, I had a tough time with it. . .It just didn't come to me as quick as it did in high school [which] I think is what was tough for me. In junior high I had to be in the, it's not dumb math, but it's like the slow learning math class. I didn't think I was—should be in that class because I was above everyone in that class but I was lower than the normal student. . .And then [in college] my test scores were higher. I mean that made me happy. I'm good now. It makes sense to me now.

Student 19 described his shortcomings in high school mathematics.

> It was just that it was making my brain work harder than I thought it could, you know. That's how I felt in high school. . .Another thing was homework. A big deal. I just—I neglected to ever do it.

He also commented on his mathematical success in his current job training program, "[It is] actually coming very easy to me [on the job]."

Overall, five of the nine non-Hands-On Equations students, Students 11, 12, 13, 15, and 16, were deemed confident. These five students had at least two Xs in the confident categories.

In addition to the two students who expressed both non-confidence and confidence, Students 11, 12, 13, 14, and 15 displayed a sense of accomplishment (Category 3). A sampling of the comments from Students 11, 12, 13, 14, and 15 shows their sense of accomplishment. Student 11: "My grades reflected that [I was good at mathematics] and it came easy to me." Student 12: "Math was one of my things I could do. . .I always had As in math classes and my ACT score was actually high. . .The first time I took it, I got a 31 on math." Student 13: "On like all the standardized tests, I always scored really high in algebra." Student 14 described the role of hard work in accomplishing mathematical success, "I caught on after like the first week because I went in and spent time with her and had her help me one-on-one." Student 15 offered, "I never really struggled with it (mathematics). . .In most of my classes I found myself explaining it to other people more than I had it explained to me."

Students 11, 12, 13, 15, and 16, as well as the two confident/non-confident students, expressed feelings of confidence (Category 4). Student 11: "It just came relatively easy to me probably because I enjoyed it a lot. It (algebra) was one of my favorite kinds of math." Student 12: "I would pretty much get it right away. . .Everyone asked me how to do it because I would know how to do it." Student 13: "For me, now it's easy. . .At this point, I'm good at algebra." Student 15: "I never had a problem with it. I thought it was easy. . .I never really struggled with it. . .I picked it up easily." Student 16: " I found it actually fairly easy."

A sampling of the comments from these same five students, Students 11, 12, 13, 15, and 16, displays their understanding of mathematics (Category 5). One of the confident/non-confident students, Student 17, expressed understanding as well. Student 11 understood algebra at the beginning of her first algebra class.

> *Interviewer*: *Did you understand algebra in that first class?*
>
> Yes.
>
> *Interviewer: At what point did you understand it?*
>
> I would say at the beginning.

Student 12: "I understood math better than some people." Student 13 answered, "Yeah," to the question about understanding algebra. He added, "[I] never really felt like totally lost in those subjects (Calculus 1, Calculus 2, and Statistics)." Student 15: "I picked it up easily." Student 16: "I understood most of it." He went on to comment on his understanding, "[A] positive would probably be when I first kind of started understanding it and was actually being able to help other people and showing them how I understood it and maybe explained it a little more." Student 17 thought that understanding was a positive, "Positives. Probably in college when it just made sense to me."

Student 14 was classified as neither confident nor non-confident because she thought mathematics was hard but she also had a positive experience in a mathematics class in junior college which bolstered her sense of accomplishment. She judged mathematics to be hard because, as she explained, "Math is not one of my strong points." Later she expressed her sense of accomplishment in her algebra course in college, "It will take a while, but eventually I will get it."

Student 18 definitely had no confidence in her mathematical abilities. She expressed her feelings that mathematics was inaccessible and pointed out her lack of accomplishment in mathematics. She explained her feelings.

Math is not my thing. . .I know I'm not good at math and

> it wasn't my strong subject. I felt kind of dumb at times because I wasn't in the smart math classes. So I think all around, that was just a big negative.

She added, "I'm just not very good with numbers and all that stuff. All my friends were in the class with all the high-tech stuff and I didn't know even how to say the word trigonometry."

The summarization of the non-Hands-On Equations data in Table 6 was done with the nine students. With the three categories in the "confident" area and nine students, there were 27 opportunities to denote confidence, and the two categories in the "non-confident" area yielded 18 opportunities for non-confidence. The results revealed 20 out of 27 confidence markings (74%) and 6 out of 18 non-confidence markings (33%) for the non-Hands-On Equations group.

Comparison of composite data results. Research question three asked if there was a difference in present mathematics self-efficacy between students taught with Hands-On Equations and those who did not experience these teaching materials. The composite results from the data are summarized in Tables 7 and 8.

In Table 7, the data show that in the Hands-On Equations group, 78% of the students termed algebra "easy" while 71% of

Table 7

Composite Self-Efficacy Comparisons for Both Student Groups in Regards to Difficulty of Algebra and Percent of Marked Xs on Tables 5 and 6

	H-O Eq	Non-H-O Eq
	Difficulty of Algebra	
Hard	11%	29%
Easy	78%	71%
Neither	11%	0%
	Percent of Xs Marked on Tables 5 and 6	
Non-Confident(X)	25%	33%
Confident (X)	57%	74%

Note. H-O Eq = Hands-On Equations group; Non-H-O Eq = Non-Hands-On Equations group; Non-Confident (X) = Non-confident responses out of the non-confident possibilities on Tables 5 or 6; Confident (X) = Confident responses out of the confident possibilities on Tables 5 or 6.

the students from the non-Hands-On Equations group called algebra “easy.” By comparison, 11% of the Hands-On Equations group termed algebra “hard” while 29% of the non-Hands-On Equations group labeled algebra “hard.”

Combining all of the Xs from Tables 5 and 6 in the respective categories of self-confidence is one way to view the data in a composite manner; this compilation is also presented in Table 7. Confident responses out of the confident possibilities and non-confident responses out of the non-confident possibilities for each group were used to compute the percents. The Hands-On Equations group confidence was 57%; the group confidence for the non-Hands-On Equations group was 74%. The Hands-On Equations group non-confidence was 25%; the non-confidence for the non-Hands-On Equations group was 33%.

Another way to think about the data is to look at the overall percent of students who were labeled as either confident, non-confident, neither confident nor non-confident, or both confident and non-confident. This approach yields the third set of numbers which are included in Table 8. In this study, 60% of the Hands-On Equations group made confident comments; 56% of the non-Hands-On Equations group felt confident. By comparison, 30% of the Hands-On Equations

Table 8

Percent of Students Judged on Feelings of Confidence for Both Student Groups

	H-O Eq	Non-H-O Eq
Non-Confident	30%	11%
Confident	60%	56%
Neither	10%	11%
Both	0%	22%

Non-Confident = Students in the respective group who were labeled non-confident; Confident = Students in the respective group who were labeled confident; Neither = Students in the respective group who were labeled neither confident nor non-confident; Both = Students in the respective group who were labeled both non-confident and confident at various stages of their academic careers.

group seemed non-confident and 11% of the non-Hands-On Equations group were non-confident. Another 10% and 11% were considered neither confident nor non-confident in the Hands-On Equations group and the non-Hands-On Equations group, respectively. None of the Hands-On Equations group was labeled as both confident and non-confident but 22% of the non-Hands-On Equations group expressed both confident and non-confident comments.

An interesting difference in initial confidence in doing basic algebra emerged in the interviews with the only two students who scored above 30 on their ACT mathematics tests. Student 2 was in the Hands-On Equations group and earned a score of 32; student 12 was in the non-Hands-On Equations group and achieved a score of 31. Student 2 recalled an observation he had made in his first algebra class.

> I remember some kids struggled with the idea of, you know, I want to just take away these two *y*s but why does this change these four *y*s over here and that was all just really basic to me. And I remember that was one of the big struggles in my class. . . I remember a couple of other people in my class understood it real well, too. So I mean, they weren't struggling either.

When asked who those two students were, he recalled their names and the fact that they were both in the Hands-On Equations class. By contrast, Student 12 expressed an initial lack of self-confidence when she arrived in her first algebra class.

> At first I was kind of like nervous about it. . . I finally started understanding everything. . .About midway through the first semester to the end of the semester, I finally understood it. . . You know, solving equations, you know, both sides. Getting to one common answer.

The two highest achieving students as determined by their ACT scores differed in their memories about their initial understandings of algebra. The Hands-On Equations student felt confident and comfortable immediately in his first algebra class whereas the non-Hands-On Equations student was apprehensive and understood algebra later in the semester.

Other Responses Related to Self-Efficacy

Found in Tables 9 and 10 are the student responses to protocol questions 6 and 11. Protocol question 6 asked if algebra was “hard” or “easy.” These labels are included in Tables 9 and 10. Students 10, 18, and 19 were not asked

Table 9

Hands-On Equations Student Responses When Asked if Algebra is Easy or Hard and When Asked if They Are Good or Not So Good at Mathematics

Student	GPA	Easy or Hard	Good or Not So Good
1	4.00	Easy	G
2	3.84	Easy	G
3	3.43	Easy	G
4	3.06	Easy	G[c]
5	2.97	Easy	SA[d]
6	2.94	Hard	NG
7	2.89	Easy	G
8	2.70	Easy	NG
9	2.64	Neutral[a]	A
10	2.13	NA[b]	NG

Note. G = good; SA = strong average; A = average; NG = not so good. [a]Student answered "not easy" and "not so hard." [b]Not applicable because student did not take an algebra class. [c]Student qualified his response as applying to algebra. [d]Student qualified his response as applying to "basic stuff."

Table 10

Non-Hands-On Equations Student Responses When Asked if Algebra is Easy or Hard and When Asked if They Are Good or Not So Good at Mathematics

Student	GPA	Easy or Hard	Good or Not So Good
11	4.00	Easy	G
12	3.88	Easy	G
13	3.49	Easy	G
14	3.44	Hard	A
15	3.28	Easy	G
16	2.93	Easy	A[b]
17	2.75	Hard	G[c]
18	2.58	NA[a]	NG
19	2.14	NA[a]	A

Note. G = good; A = average; NG = not so good.
[a]Not applicable because student did not take an algebra class. [b]Student qualified his response as applying to "basic stuff." [c]Student qualified her answer as "good *now*."

protocol question 6 since they did not take algebra in junior high school, high school, or college. Protocol question 11 asked students to decide if they were “good” or “not so good” at mathematics.

Of the nine Hands-On Equations students who were asked protocol question 6, seven (78%) termed algebra “easy.” One student (11%) thought that algebra was “hard.” One student (11%) gave a neutral response to protocol question 6.

Of the nine non-Hands-On Equations students, two were not asked protocol question 6 because they had never enrolled in an algebra class and the question was thus not applicable to them. For the remaining seven non-Hands-On Equations students, the compiled results for protocol question 6 showed that five (71%) termed algebra "easy" while two (29%) felt that algebra was "hard."

All ten of the Hands-On Equations students were asked protocol question 11 since it applied to mathematics. Five students (50%) claimed to be good at mathematics. One (10%) judged himself to be “strong average” in dealing with “basic stuff.” One (10%) said that she was average in mathematics. Three (30%) responded to protocol question 11 by saying that they were not so good at mathematics.

All nine of the non-Hands-On Equations students were asked protocol question 11 since it applied to mathematics. Five students (56%) claimed to be good at mathematics. Three (33%) judged themselves to be average. One (11%) said that she was not good in mathematics.

Attitude and Achievement

Attitude

Attitude data were gathered from student responses to protocol questions 5, 6, 7, 9, 10, and 15. Other attitudinal comments that were made throughout the interviews were also noted. Data for student attitude as revealed in protocol question 9 that asked students to rank order 10 subject areas including mathematics are compiled in Tables 11 and 12. The data for students who experienced Hands-On Equations are in Table 11; the data for students who did not experience Hands-On Equations are found in Table 12. During the interviews, some students volunteered that they enjoyed/liked or disliked mathematics or algebra. These results are also recorded in Tables 11 and 12. As a group, the students who experienced Hands-On Equations gave mathematics an average ranking of 4.90 on a scale of 1 to 10 with 1 indicating that mathematics was the most favorite of the ten listed subjects and 10 assigned to the

Table 11

Hands-On Equations Student Rankings Assigned to Mathematics and Whether or Not the Students Liked Mathematics

Student	GPA	Mathematics Rankings	Enjoyed/Liked Mathematics or Algebra
1	4.00	2	Yes
2	3.84	4	Yes
3	3.43	3	Yes
4	3.06	1	Neutral
5	2.97	7	Yes[a]
6	2.94	10	Neutral
7	2.89	3	Yes[a]
8	2.70	7	Yes[a]
9	2.64	4	Neutral
10	2.13	8	No[b]
Mean	3.06	4.90	

Note. Mathematics rankings ranged from 1 (high) to 10 (low). Neutral = Students whose comments were deemed neutral, or neither liking nor disliking mathematics. [a]Students who specifically liked algebra. [b]No = Student who specifically expressed dislike of mathematics.

Table 12

Non-Hands-On Equations Student Rankings Assigned to Mathematics and Whether or Not the Students Liked Mathematics

Student	GPA	Mathematics Rankings	Enjoyed/Liked Mathematics or Algebra
11	4.00	2	Yes[a]
12	3.88	2	Yes
13	3.49	8	No[b]
14	3.44	5	Neutral
15	3.28	8	No[b]
16	2.93	5	Neutral
17	2.75	3	Neutral
18	2.58	10	No[b]
19	2.14	9	Neutral
Mean	3.17	5.78	

Note. Mathematics rankings ranged from 2 (high) to 10 (low). Neutral = Students whose comments were deemed neutral, or neither liking nor disliking mathematics. [a]Student who specifically liked algebra. [b]No = Students who specifically expressed dislike of mathematics.

subject that was the least favorite. The non-Hands-On Equations group average ranking was 5.78. These data indicate that the Hands-On Equations group favored mathematics noticeably more than the non-Hands-On Equations group did. One student from the Hands-On Equations group assigned mathematics the highest possible ranking of 1 whereas none of the students in the non-Hands-On Equations group gave mathematics the highest ranking. Each group had one student who felt that mathematics deserved a “least favorite” ranking of 10.

Two of the non-Hands-On Equations students, Students 13 and 15, assigned mathematics a ranking of 8. Their dislike for mathematics contrasts with their ability in the subject; Student 13 earned an ACT mathematics score of 29 while Student 15 achieved an ACT mathematics score of 25. When asked why he ranked mathematics the way he did, Student 13 explained the source of his feelings.

> I never found it very fun. . . and there have been times like where math was really hard so it’s kind of like I don’t want to do math. . . I know algebra now is easy but . . . at the time it was really hard so it’s not like I would look forward to doing it.

He confirmed his opinion later in the interview when he gave a negative experience in mathematics. “One negative is it’s not much fun.” His negative attitude about mathematics was apparent when he responded to a question about the main idea of algebra.

> *Interviewer: What was the main idea of algebra? What was the point?*
>
> The point? It’s required. There wasn’t a point.

Student 15 gave his reasoning for assigning a ranking of 8 to mathematics; “I wouldn’t say it’s because it’s that difficult, just because it wasn’t that interesting to me.”

For these same two students, attitude and interest in mathematics appeared to be linked to the perceived applicability of mathematics. Each of them responded to the question that asked them to recall two positive experiences in mathematics with a story that featured an application. Student 13 offered the following.

> We used to always bother [one of our mathematics teachers] about never being able to apply like geometry. And one time me and [my friend] were flying kites and we knew how long our kite string was, and we found a spot directly underneath the kite and [my friend] counted out

> the steps from me to him because he was underneath the kite and I was holding the string, so we knew two sides of the triangle and we figured out how high off the ground it was. And I think I emailed [another of our mathematics teachers] about it and told him that we actually used something we learned in high school. That's one positive.

Student 15 told this story about his summer job as an example of a positive experience.

> I guess just today I used it a little bit because we dumped a bunch of brush at the dump. . . I kind of looked at the sheet and figured out how many tons we had because it's not actually on there with, you know, what we weighed in as we left and we came back and how much it cost.
>
> *Interviewer: So how was that a positive for you?*
>
> Well, I just kind of explained it to the guy that was driving. It was like, oh, just so he knew. So it made our job a lot easier I guess, more interesting. We knew how much we loaded on there.
>
> *Interviewer: So, it made it more interesting?*
>
> Yeah. . . just being able to do that . . . satisfaction, I guess.

Another indicator of student attitude toward mathematics in this study was expressed enjoyment or liking of mathematics—or voiced dislike for mathematics. Students volunteered this information at various times during the interviews; these comments were noted during the thorough readings of the interview transcripts.

Within the Hands-On Equations group, 6 of the 10 students shared comments about enjoying or liking mathematics. The six students were Students 1, 2, 3, 5, 7, and 8. Student 1 said, "I've just always enjoyed it. It's been a subject—unlike any other subject—I've never really dreaded doing the homework when I've had it. . .I can sincerely say I enjoy math." From Student 2, "I have always liked math a little more because it's a—it's concrete, you know, something is happening and you can see why as opposed to like English." Student 3 offered: "I've always enjoyed math." Student 5 made a positive comment about algebra when he was presented with the six equations to solve, "I like this stuff." Student 7 affirmed, "I liked algebra a lot. Math would be one of my favorites. . .Out of all the hard core subjects, I liked math the best." While solving the six equations, she commented, "This is fun." Of these 6 students, 3 specifically mentioned liking algebra only. Thus 50% of the students in this group who liked mathematics specifically mentioned enjoying algebra. Student 8 volunteered a representative quip, “I like algebra and I don’t like any other math. That’s my big comment!”

Student 10 of the Hands-On Equations group was the only student in that group who specifically mentioned a dislike of mathematics. He said, “I didn’t really like math, but I liked it better than English, I guess.”

Two of the nine non-Hands-On Equations students expressed enjoyment of mathematics. When Student 11 was asked to describe her experiences when she got to her first algebra class, she shared, “I enjoyed it a lot. I found it to be tough but I enjoyed it a lot. . .It (algebra) was one of my favorite kinds of math.” Student 12 commented on mathematics, “I’ve always enjoyed it.”

Three of the nine non-Hands-On Equations students expressed a dislike for mathematics. Those students were Students 13, 15, and 18. The dislike that Students 13 and 15 have for mathematics permeated their many comments about mathematics not being interesting or fun. Also, they both gave mathematics a rank of 8 out of 10. Student 18 clearly stated, “I just don’t like it.”

In contrast to 60% of the Hands-On Equations students, only 22% of students in the non-Hands-On Equations group gave any indication of enjoying or liking mathematics. Only 10% of the Hands-On Equations students described a dislike of mathematics whereas 33% of the non-Hands-On Equations group disliked mathematics.

Achievement

Data for student achievement are compiled in Tables 13 and 14. The data for students who experienced Hands-On Equations are in Table 13; the data for students who did not experience Hands-On Equations are found in Table 14. Students were rank-ordered within each of the two groups by their high school overall grade point averages. Included in these tables are the grade point averages (GPAs), ACT mathematics scores, and scores on the six equations the students were asked to solve at the end of the interviews. The six equations were scored dichotomously; each equation was either completely correct for a score of 1 or incorrect for a score of 0. As a group, the Hands-On Equations students had a lower mean GPA of 3.06 and a lower mean ACT mathematics score of 20.63 when compared to the non-Hands-On Equations group which had group means of 3.17 for the GPA and 23.43 for the ACT. Specific achievement

Table 13

GPAs, ACT Scores in Mathematics, and Equation-Solving Scores for Hands-On Equations Students

Student	GPA	ACT Score in Mathematics	Equations Score
1	4.00	28	6
2	3.84	32	6
3	3.43	22	6
4	3.06	NA	5
5	2.97	17	6
6	2.94	16	5
7	2.89	17	5
8	2.70	17	2
9	2.64	16	2
10	2.13	NA	0
Mean	3.06	20.63	4.30
Mean Percent for the Equation Solving			72%

Note. The total possible for the equation solving was six.
Equations Score = total score on the six equations. NA = no score for the ACT.

Table 14

GPAs, ACT Scores in Mathematics, and Equation-Solving Scores for Non-Hands-On Equations Students

Student	GPA	ACT Score in Mathematics	Equations Score
11	4.00	27	6
12	3.88	31	5
13	3.49	29	6
14	3.44	16	0
15	3.28	25	6
16	2.93	21	4
17	2.75	15	5
18	2.58	NA	0
19	2.14	NA	0
Mean	3.17	23.43	3.56
Mean Percent for the Equation Solving			59%

Note. The total possible for the equation solving was six.
Equations Score = total score on the six equations. NA = no score for the ACT.

in solving one-variable linear equations contrasted with the ACT and GPA results. The Hands-On Equations group mean was 4.30 out of a possible 6.00; the non-Hands-On Equations group mean was 3.56. This data reflect a success rate or accuracy percent on solving the six equations of 72% for the Hands-On Equations group versus 59% for the non-Hands-On Equations group.

CHAPTER 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Introduction

"Our [qualitative researchers'] small samples and always too-brief opportunities to observe preclude us from reporting authoritative correlations, but careful reporting of what we actually observe—even in single instances—is an important contribution from our work" (Wolcott, 1994, p. 33). Wolcott (1990) recommended working "toward a conservative closing statement that reviews succinctly what has been attempted, what has been learned, and what new questions have been raised" (p. 56). This closing chapter attempts to follow the advice of this experienced qualitative researcher and author.

Summary

Many students flounder when confronted with algebra for the first time in a formal class in the eighth or ninth grade because they are expected to assimilate algebraic ideas and skills in a very short time (Greenes & Findell, 1999; Von Rotz & Burns, 2002). National and state organizations such as the

National Council of Teachers of Mathematics (NCTM, 2000) and the Kansas State Board of Education (2004) recommended teaching algebraic concepts during elementary school. Earlier national recommendations for an algebra strand in elementary school also exist (Educational Testing Service and the College Board, 1990; NCTM, 1989, 1992). Even though professional groups have recommended that algebra be included in kindergarten through grade 8, this has not happened consistently (Greenes & Findell, 1999).

Algebra is distinctly different from arithmetic; algebra focuses on general relationships and has been termed the generalization of arithmetic (Costello, 1993; Esty, 1999; MacGregor & Stacey, 1999; Saul, 2001; Tierney & Nemirovsky, 1997). Children as young as sixth grade are able to go from the specific to the general by discerning patterns and then generating helpful formulas (Herbert & Brown, 1997).

Algebra as a mathematical topic in elementary school looks different from the more formal algebra of middle school or high school but involves the same basic understandings (NCTM, 2000). NCTM's Standard 2: Algebra promoted student understanding of patterns, relations, and functions; the use of algebraic symbols; the use of mathematical models to represent and understand quantitative relationships; and the analysis of change in various contexts. At both the state (Kansas State Board of Education, 2004; Alaska Department of Education & Early Development, n.d.; Arizona Department of Education, 2003) and national (NCTM, 2000) levels, specific algebraic expectations vary depending on the grade level.

The National Assessment of Educational Progress (NAEP) assessed important, informal algebraic concepts related to patterns and relationships (Kenney & Silver, 1997). The 1992 NAEP assessed fourth graders' algebraic thinking involving patterns of figures, symbols, or numbers. Fourth graders could reason with simple patterns but had more trouble with complex patterns and explaining their mathematical reasoning about patterns. A close examination of the NAEP algebra and functions question data for fourth-grade students from *The Nation's Report Card* (National Center for Education Statistics, n.d.) revealed that a majority of the students could answer procedural knowledge questions correctly but stumbled when required to deal with conceptual-understanding and problem-solving questions. This information confirms the pattern for American students of performing well on computational tasks but not achieving as well on problems that demand deeper mathematical understanding (Kilpatrick, Swafford, & Findell, 2001).

Data from the state of Kansas (Center for Educational Testing and Evaluation, n.d.a; Center for Educational Testing and Evaluation, n.d.b) reported achievement levels of fourth-grade students. These data indicated an improvement in algebraic knowledge for Kansas fourth graders from 2000 to 2005.

The *Third International Mathematics and Science Study* (TIMSS) was a large-scale international study that revealed the comparative inadequacies of algebra learning in American schools (U.S. National Research Center, 1996). For example, United States students in the eighth grade study arithmetic, fractions, and a small amount of algebra in contrast to both Japan and Germany whose students receive thorough exposure to both algebra and geometry.

Kieran (1992) included two areas that impact student learning in algebra: the content and the students. Various authors make the case that the content of algebra is inherently difficult because algebra is both a language and an abstract system with specific rules that are difficult to learn (Esty, 1999; Hatfield, Edwards, Bitter, & Morrow, 2005; Usiskin, 1996; Von

Rotz & Burns, 2002). Kieran (1989) observed that the language aspect of algebra is difficult for high school students to decipher. Kieran's (1992) analysis of the research related to the learning of algebra supported her overall conclusion that students do not understand the more difficult structural aspects of algebra. Bruner (1960) and Rittle-Johnson and Alibali (1999) also emphasized the importance of helping students understand the structure of the subject.

Kilpatrick et al. (2001) confirmed that many students have difficulty in making the transition from school arithmetic to school algebra for various reasons including the symbolism of algebra. Rubenstein and Thompson (2001) asserted that students who do not master the standard symbolism of mathematics will be hindered at some point in their mathematical careers. Oftentimes, students do not understand the equality symbol as an indicator of equality (Behr, Erlwanger, & Nichols, 1976; Erlwanger & Berlanger, 1983; Kieran, 1981; Saenz-Ludlow & Walgamuth, 1998).

A lack of opportunity to learn algebra may be another real issue for students; the NAEP (National Center for Education Statistics, n.d.) data bank revealed how infrequently fourth-grade teachers addressed algebra and functions. When teachers do teach the subject, the Third International Mathematics and Science Study (TIMSS) revealed that students often experience traditional teaching methodologies rather than the reform curriculum suggested by research (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). Carraher, Schliemann, Brizuela, and Earnest (2006) concluded that students' difficulties with algebra may more closely relate to teaching techniques rather than a developmental inability. One reason for this lag in appropriate teaching may be that United States mathematics teachers rarely have time to teach any subject in depth because they are expected to teach such a wide range of subjects (Lane, 1996). This breadth instead of depth is obvious in the textbooks and curriculum and makes a difference in student learning (Schmidt, 1996). An often-quoted accusation described the United State's curriculum, textbooks, and teaching as "a mile wide and an inch deep" (Schmidt, 1996, ¶ 5).

Two distinct theories over the past 100 years have shaped the discussion on learning. Behaviorism deals with externally observable events, instructional manipulations and outcome performance, whereas a cognitive approach to learning includes internal factors such as learning processes and existing learner characteristics (Mayer, 1999). An example of behaviorism in mathematics education is the use of flash cards when teaching math facts (Hohn, 1995). Constructivism is a cognitive learning theory that emphasizes that understanding must be constructed by the learner (Lemlech, 2002). Constructivism has become the accepted theoretical position among mathematics education researchers (Battista, 1999; von Glasersfeld, 1995). Others (Hohn, 1995; Goldin & Shteingold, 2001) advocated for a unified theory that includes both schools of thought. Such an inclusive educational philosophy would include both behaviorism and constructivism and would value both skills acquisition and complex problem solving (Goldin & Shteingold, 2001).

When learning mathematics, children either retrieve solutions from memory or revert to more time-consuming alternative strategies that make sense to them (Siegler, 1998). Attempted recall with limited conceptual understanding leads to problems in the learning of mathematics. In algebra, Siegler asserted that superficial understanding is exemplified by students who merely manipulate the algebraic symbols without understanding any real-world applications. Such students may make incorrect extensions of correct rules and generalize inaccurately. Current thinking on the cognitive development of

11- and 12-year-olds questions Piaget's idea of explicit general developmental stages (Flavell, 1992; Siegler, 1998). Contemporary developmentalists believe that cognitive development is more balanced with both general stagelike attributes and specific properties that relate to particular content areas (Flavell, 1992). Prior content knowledge influences what people are able to learn (Siegler, 1998). Mayer (1999) and Bruner (1960) supported the idea that skills needed to solve mathematics problems can be taught regardless of a student's age.

Manipulative materials are objects that can be handled by the learner (Kennedy, 1986). Manipulatives have been shown to help children move from the concrete level to the abstract level (Hartshorn & Boren, 1990). Hartshorn and Boren found that a transition stage between these two levels is crucial. Teachers must carefully structure the use of the connecting or pictorial intermediate stage in order for students to make the connection (Hartshorn & Boren, 1990; Sa'ar, n.d.; Witzel, Smith, & Brownell, 2001). Mental imagery formed by handling manipulative materials helps students understand mathematical concepts (Bell & Tuley, 2006; Kennedy, 1986; Moyer, 2001).

Manipulative materials should be used at all school levels (Kennedy, 1986). Stewart (2003) noted that the increase in abstraction in mathematics in elementary grades often coincides with the decrease in the use of manipulatives. Much research exists on the importance of manipulatives at the elementary school level but there is little information involving manipulatives at the middle and high school levels (Weiss, 2006).

Henry Borenson (1994) created manipulative materials, the *Hands-On Equations Learning System*, to support the learning of algebra for students as young as 8 years old. Borenson claimed that the system (hereinafter referred to as Hands-On Equations) imparts important mathematical content, promotes mathematical interest, and heightens student self-esteem. These materials were specifically designed to meet the algebraic needs of teachers and students in the elementary grades. Borenson's materials have been available since the early 1990s, but few research studies have been done to explore the value of Hands-On Equations.

Four studies dealt with Hands-On Equations. The 123 sixth graders in Barclay's (1992) study were taught five lessons with Hands-On Equations. Posttest results showed that 100% of the students demonstrated at least 80% mastery on at least two of three posttests. Leinenbach and Raymond (1996) worked with eighth graders. When these students took a standardized algebra test, their performance far exceeded expectations. The students also expressed more positive attitudes about algebra when working with the manipulative materials in Hands-On Equations. Busta (1993) studied the impact of Hands-On Equations on 335 middle school students in grades 6, 7, and 8. The students were taught one lesson per week for seven weeks. The sixth graders who experienced Hands-On Equations did significantly better on the posttest than the control group did. Virtual and physical manipulatives for adding fractions and balancing equations were included in Suh's (2005) research. Two classrooms with a total of 36 students were taught four lessons with virtual manipulatives in one content area (fractions or algebra) and then experienced four lessons with physical manipulatives on the other topic (fractions or algebra). The physical manipulatives used when teaching the balancing of equations was Hands-On Equations. Students in the virtual manipulative fraction treatment group performed statistically better than the students who worked with the physical manipulative fraction circles.

There was no statistically significant difference between the virtual and physical algebra methods.

The advantages of Hands-On Equations have been reported by others (Borenson and Associates, n.d.b; Carnopis, 1987; Ghazi, 2000). Borenson claimed that third, fourth, and fifth graders were taught to solve equations such as $4x+2=3x+9$ during live demonstrations at over 1000 workshops during a recent 10-year period. He explained that the game-like format is interesting to students (Borenson, 1994) and gives them confidence and self-esteem that allows them to feel good about algebra (Carnopis, 1987). Ghazi (2000) observed and reported on the enthusiasm of 10-year-olds when working with Hands-On Equations. She wrote that British mathematics experts were astounded that such young children could work problems usually reserved for bright 12-year-olds or average 14-year-olds in Britain.

Research has shown that memory and learning are related topics. Memory is better understood when partitioned rather than considered as one unit (Terry, 2006). Terry explained that generalizations about memory as a whole are not valid but descriptors of particular forms of memory are accurate. One partitioning involves two major memory components: short-term memory and long-term memory. Long-term memory is further divided into three types of memory—procedural, semantic, and episodic (Tulving, 1985). A second partitioning of memory involves three stages of memory (Terry, 2006). Those stages are encoding, storage, and retrieval; any one of these stages could contain problems that lead to forgetting. Effective retrieval from episodic memory, the autobiographical or personal memory system, depends on factors such as the distinctiveness of the memory and retrieval cues. One explanation for why distinctive events are retrieved better is that their retrieval cues are uniquely linked with a single memory. Cues that were encoded with the recalled item or event are good retrieval cues. A third partitioning of memory deals with processes of memory and has two sub-categories, depth of processing and transfer-appropriate processing (Terry, 2006). The importance of depth of processing is widely accepted as leading to comprehension of material. Transfer-appropriate processing links the encoding and retrieval stages for optimum remembering. Terry emphasized that the three approaches to memory were complementary rather than exclusionary.

Bartlett's *Remembering: A Study in Experimental and Social Psychology* has been continually cited in the literature since it was first published in 1932 (Johnston, 2001). Bartlett (1932) described "every human cognitive reaction—perceiving, imaging, remembering, thinking and reasoning—*as an effort after meaning*" (p. 44). Johnston (2001) linked Piaget and Bartlett by their common use of the schema concept. Schema involves the active organization of past reactions or past experiences (Bartlett, 1932). Bartlett asserted that remembering is more construction than mere reproduction. He also explained that attitude impacts all of memory.

Several variables on long-term memory for knowledge learned in classrooms were delineated by Semb, Ellis, and Araujo (1993). They included the "degree of original learning, the tasks to be learned, characteristics of the retention interval, the method of instruction, the manner in which memory is tested, and individual differences" (p. 305). Howe(2000) emphasized the importance of knowing the initial degree of learning in order to interpret later assessments of long-term retention. Terry (2006) stated that the initial level of acquisition impacts retention. Bahrick, Bahrick, and Wittlinger (1975) made the case for the non-laboratory approach to the study of memory. They found that people could remember classmates' names and faces for almost 50 years. They attributed this long recall to distribution of practice and overlearning of the naturalistically learned material. Bahrick (1984) looked at knowledge learned in school and found that there is a semi-permanent nature of unrehearsed knowledge. Other researchers (Conway, Cohen, & Stanhope, 1991; Semb, Ellis, & Araujo, 1993) also found that students remembered classroom learning over long periods of time.

Hypermnesia is the "abnormally vivid or complete memory or recall of the past" (Woolf, 1981, p. 558). Bahrick and Hall (1993) concluded that hypermnesia may be apparent when assessment tests cover a stable body of content knowledge.

Research has studied the role of self-efficacy on students' learning. Bandura (1986) defined self-efficacy as the self assessment of one's capability to succeed to a certain level in specific subject areas. Hohn (1995) and Pajares and Miller (1995) also explained that self-efficacy is specific to particular tasks or certain domains. Of the major sources of developing self-efficacy, personal performance accomplishments contributed the most influence (Lent, Lopez, & Bieschke, 1991). Lent et al. recommended educational interventions for students with low mathematics self-efficacy. They suggested that numerous successful structured mastery experiences would contribute to a more positive mathematics self-efficacy in students. Self-efficacy may impact important outcomes such as career choice (Hackett & Betz, 1989), and effort and persistence in the face of obstacles (Bandura, 1986; Borget & Gilroy, 1994). More recent research (Pietsch, Walker, & Chapman, 2003) confirmed that self-efficacy in mathematics predicts future performance in mathematics.

Many researchers have studied the impact of attitude on student learning of mathematics. A favorable attitude toward mathematics would lead to moving toward behavior whereas avoidance behavior is associated with a negative attitude (Mager, 1968). Mager asserted that teachers should influence students to develop a favorable attitude toward a subject in order to maximize the possibility of remembering, using, and learning more about that subject in the future.

Ma and Kishor (1997) conducted a meta-analysis on the relationship between attitude toward mathematics (ATM) and achievement in mathematics (AIM). The overall mean effect was "statistically significant but not strong for educational practice" (p. 39) and the effect size for the causal relationship for ATM (cause) and AIM (effect) was insignificant and deemed to have no practical implication. They concluded that current attitude measures do not reflect true attitudes and recommended that researchers should refine these assessment tools for better future results. Ma and Kishor also believed that the ATM-AIM relationship may be impacted the most during the junior high school years. Singh, Granville, and Dika (2002) found that attitude is influential in explaining mathematics achievement variations.

Moyer and Jones (1998) claimed that the use of manipulatives has "the potential to improve student attitudes and student intrinsic motivation" (p. 35). Students used manipulatives as learning tools for constructing meaning. Moyer and Jones advocated the use of manipulatives as often as other mathematical tools such as rulers and protractors.

The purpose of this study was to examine the perceptions of high school graduates who experienced the mathematical materials from Hands-On Equations when the students were in the sixth grade. The investigation also included the perceptions of students who did not experience Hands-On Equations during their sixth-grade year. Four research questions were addressed.

1. For the students who experienced Hands-On Equations, what is the perceived value of these materials?

2. Did the Hands-On Equations lessons create student perceived differences in subsequent learning in algebra classes for students taught with Hands-On Equations?
3. Is there a difference in present mathematics self-efficacy between students taught with Hands-On Equations and those who did not experience these teaching materials?
4. Are there other differences related to (a) student attitudes toward mathematics, (b) student achievement in mathematics, and (c) student ability to solve simple linear equations between students taught with Hands-On Equations and students who were not?

The participants for this research were college-age students who had graduated from high school in a small, public school district in eastern Kansas. Of the 19 students who were interviewed, 10 had experienced the manipulative learning materials, Hands-On Equations, when they were in the sixth grade. Ten of the students were male and nine were female. All students in the study had attended schools in the same district from 6th grade through 12th grade.

Data collection was achieved by interviewing the 19 students. The interviews were conducted by this researcher (16 interviews) and another person qualified to do qualitative research (3 interviews). The interviews were held in the summer of 2005 and were structured by using protocol questions. The last question during the interviews asked participants to solve six one-variable linear equations. Tape recordings and transcriptions of the interviews were made. Additional demographic information about each student was gathered from high school records.

The transcriptions, the six equations worksheets, and the students' high school records constituted the data. The work of Esty and Teppo (1994) served as an organizational guideline to review the self-confidence aspects of the data. Attitude results were based on the student responses during the interviews. The responses included the rankings assigned to mathematics by the students as well as student comments about liking or disliking mathematics. The data on GPAs and ACT scores were reviewed (a) for any discrepancies between the two groups of students and (b) to see if there were any relationships between student interview responses and the data from the ACT and GPA scores. The results gleaned from the data were reported in narrative form and in tabular form.

The results of the analysis of the data were divided into five sections. The first three of these five sections dealt only with the data associated with the students who experienced Hands-On Equations when they were in the sixth grade. The first section described student memory of Hands-On Equations. The second through the fifth sections corresponded to the four research questions.

All but one of the students who learned with Hands-On Equations remembered the experience. Most remembered something without the physical prompt of seeing the student materials (the laminated mat and the manipulative pieces). The four students who had much recall shared vivid and detailed memories. When asked to recall any reactions to Hands-On Equations, they all gave answers related to academics rather than emotion. Three of these four students had the highest grade point averages (4.00, 3.84, and 3.43) and the highest mathematics ACT scores (28, 32, and 22, respectively). Other students described emotional responses. All remembered having positive reactions to the experience. As asserted by Rubin and Rubin (1995), the detailed examples and rich narratives provided by the interviews contributed to the in-depth understanding of these students' perceptions.

The perceived value of these materials was gathered when the students were asked if other sixth graders should learn Hands-On Equations and from responses to other protocol questions. Although their reasons varied, all students recommended Hands-On Equations for other sixth-grade students. The fact that one student could not specifically recall Hands-On Equations leads one to interpret his positive response to this question as only an indirect approval of Hands-On Equations. The reasons for valuing the Hands-On Equations materials included the access to foundational algebraic knowledge that would help students when they got to their first algebra class, alignment with visual or hands-on learning styles, and the promotion of student interest in mathematics.

Students were asked if Hands-On Equations made a difference for them in subsequent mathematics courses. Almost all felt that it had. Half of the students specifically credited Hands-On Equations with making a difference for them when they took their first algebra course. A large majority of the Hands-On Equations students deemed algebra to be "easy."

Several protocol questions as well as related comments throughout the interviews were used to determine student self-efficacy. Self-efficacy data were analyzed according to the self-confidence guidelines developed by Esty and Teppo (1994). The self-confidence characteristic was labeled as either “non-confident” or “confident” with two non-hierarchical, descriptive sub-categories under “non-confident” and three non-hierarchical, descriptive sub-categories under “confident.” Adaptations of their two classifications in the area of self-confidence were used in the present study. Students who were labeled non-confident may have described mathematics as inaccessible or may have expressed a lack of accomplishment. Confident students may have relayed a sense of accomplishment, feelings of confidence in their performance in mathematics, or commented on their ability to understand mathematics.

The interview transcripts were carefully read and coded for comments that indicated mathematical self-efficacy as determined by either student confidence or non-confidence with mathematics or algebra. One way to think about the data is to look at the overall percent of students who were labeled as either confident, non-confident, neither confident nor non-confident, or both confident and non-confident. Students with at least one comment in two of the three confident categories were deemed confident. In this study, 60% of the Hands-On Equations group made confident comments; 56% of the non-Hands-On Equations group made confident comments. A student who had at least one comment in either of the two non-confident categories was considered to be non-confident. By comparison, 30% of the Hands-On Equations group made non-confident comments and 11% of the non-Hands-On Equations group made non-confident comments. Using this system, some students were labeled neither confident nor non-confident, or both confident and non-confident. Another 10% and 11% were considered neither confident nor non-confident in the Hands-On Equations group and the non-Hands-On Equations group, respectively. None of the Hands-On Equations group was labeled as both confident and non-confident but 22% of the non-Hands-On Equations group expressed both confident and non-confident comments.

Confident responses out of the confident possibilities and non-confident responses out of the non-confident possibilities on tables for each group were used to compute percents. The Hands-On Equations group confidence was 57%; the group confidence for the non-Hands-On Equations group was 74%. The Hands-On Equations group non-confidence was 25%; the non-confidence for the non-Hands-On Equations group was 33%.

Other self-efficacy data showed that in the Hands-On Equations group, 78% of the students termed algebra “easy” while 71% of the students from the non-Hands-On Equations group called algebra “easy.” By comparison, 11% of the Hands-On Equations group termed algebra “hard” while 29% of the non-Hands-On Equations group labeled algebra “hard.”

The last segment of the data analysis dealt with the differences between the two student groups in three areas. Those areas were student attitudes toward mathematics, student achievement in mathematics, and student ability to solve simple linear equations.

Students were asked to rank order 10 subject areas including mathematics. As a group, the students who experienced Hands-On Equations gave mathematics an average ranking of 4.90 on a scale of 1 to 10 with 1 indicating that mathematics was the most favorite of the ten listed subjects and 10 assigned to the subject that was the least favorite. The non-Hands-On Equations group average ranking was 5.78. These data indicated that the Hands-On Equations group favored mathematics more than the non-Hands-On Equations group did. One student from the Hands-On Equations group assigned mathematics the highest possible ranking of 1 whereas none of the students in the non-Hands-On Equations group gave mathematics the highest ranking. Each group had one student who felt that mathematics deserved a “least favorite” ranking of 10.

During the interviews, some students volunteered that they enjoyed/liked or disliked mathematics or algebra. In contrast to 60% of the Hands-On Equations students, only 22% of the students in the non-Hands-On Equations group gave any indication of enjoying or liking mathematics. Only 10% of the Hands-On Equations students described a dislike of mathematics whereas 30% of the non-Hands-On Equations group disliked mathematics.

Achievement data were gathered from grade point averages (GPAs), ACT mathematics scores, and scores on the six equations the students were asked to solve at the end of the interviews. As a group, the Hands-On Equations students had a lower mean GPA of 3.06 and a lower mean ACT mathematics score of 20.63 when compared to the non-Hands-On Equations group which had group means of 3.17 for the GPA and 23.43 for the ACT.

Specific achievement in solving one-variable linear equations contrasted with the ACT and GPA results. The Hands-On Equations group mean was 4.30 out of a possible 6.00; the non-Hands-On Equations group mean was 3.56. These data reflect a success rate or accuracy percent on solving the six equations of 72% for the Hands-On Equations group versus 59% for the non-Hands-On Equations group.

Conclusions

The conclusions that follow address the four research questions of this study. Other observations and conclusions from the data follow the discussion of the four research questions.

1. The student perceptions of the Hands-On Equations experience were very positive. The sixth-grade experience was vivid enough in these students' memories 8 years later that all but one had recall. These results dealing with the ability to remember perceptions of an earlier experience are similar to those of Rippey, Geller, and King (1978) who found that student recollections of a previous state of knowledge were valid enough to use as retrospective pretests that could then be utilized to infer learning or change. Rippey et al. also explained that it was important to have the cooperation of the subjects in such a retrospective study. This study met that criterion.

Students valued their experience and all recommended that other sixth-grade students learn algebra with Hands-On Equations. The students valued their learning experience because it gave them foundational algebraic knowledge that helped them when they took their first algebra course, it aligned with their hands-on learning style, or it promoted more interest in mathematics than did the more traditional approach to teaching the subject. Many of the Hands-On Equations students remembered enjoying working with the manipulative materials. This enjoyment of Hands-On Equations is similar to the observations of the teachers in Busta's 1993 study; informal conversations with the seventh-grade teachers who implemented Hands-On Equations indicated that "students had thoroughly enjoyed working with the materials" (p. 118). Leinenbach and Raymond (1996) noted that the students in their study did not enjoy the textbook approach as much as they did the Hands-On Equations manipulatives.

2. Most of the Hands-On Equations students perceived that Hands-On Equations made a difference in later mathematics classes. Half of the Hands-On Equations students specifically volunteered that Hands-On Equations made a difference in their first algebra class. They concluded that the experience gave them the foundational knowledge that made their more formal algebra class comfortable and understandable. Several commented on the apparently similar comfort levels of their

Hands-On Equations peers while noting that some of those who did not have that experience seemed confused initially in that first algebra class.

These data support the research of others. Hartshorn and Boren (1990) found that manipulatives can play a crucial role in helping students move from the concrete level to the abstract level. Witzel, Smith, and Brownell (2001) advocated the use of manipulatives to help students understand abstraction on a concrete level. Moyer (2001) explained that the sensory experiences with manipulatives help students understand mathematical concepts. Borenson's (1994) claims of greater foundational understanding of algebra for students who use Hands-On Equations are also supported by the current study.

3. There were marginal differences in self-efficacy between students taught with Hands-On Equations and those who were not exposed to these manipulative teaching materials. Approximately the same percent of students in each group were labeled confident in mathematics. A large majority of each group termed algebra "easy." The only notable difference in the data was the fact that 11% of the Hands-On Equations group termed algebra "hard" while 29% of the non-Hands-On Equations group labeled algebra "hard."

These results are related to the findings of Lent, Lopez, and Bieschke (1991) who found that personal performance accomplishments contributed the most influence as a source of efficacy information. The non-Hands-On Equations students had higher achievement in mathematics based on their ACT scores and GPAs but little difference was apparent in the self-efficacy between the two groups. Based on the research of Lent et al., one would expect that the Hands-On Equations group would have lower self-efficacy in mathematics. Perhaps their approximately equal self-efficacy is notable. This interpretation may be supported by the fact that the Hands-On Equations students did not describe algebra as "hard" as frequently as did the non-Hands-On Equations students.

4. Differences were noted between the two groups when student attitudes were examined. Hands-On Equations students favored mathematics noticeably more than the non-Hands-On Equation group did. A majority of the Hands-On Equations students in the present study proclaimed a liking for mathematics or algebra whereas less than one-fourth of the non-Hands-On Equations students liked mathematics or algebra. The students' expressions of dislike for the subjects confirmed this situation. Only one-tenth of the Hands-On Equations students

described a dislike of mathematics whereas one-third of the non-Hands-On Equations group disliked mathematics. These results are similar to those of Leinenbach and Raymond (1996) who found that the eighth-grade students in their study expressed more positive attitudes about algebra when working with Hands-On Equations than when the same students worked with algebra textbooks. The results of the current study also relate to Ma and Kishor's (1997) results that found that the relationship between student attitude toward mathematics and achievement in mathematics may be impacted the most during the junior high school years.

5. The results indicated that the students in the Hands-On Equations group, despite lower GPAs and lower mathematics ACT scores, were better able to solve simple linear equations. The Hands-On Equations group had both a lower mean GPA and lower mean ACT mathematics score than did the non-Hands-On Equations group. The Hands-On Equations group had a mean GPA of 3.06 and a mean mathematics ACT score of 20.63. The non-Hands-On Equations group had a mean GPA of 3.17 and a mean mathematics ACT score of 23.43. The conclusion from this data is that the non-Hands-On Equations group generally achieved above the Hands-On Equations group. In contrast, the

Hands-On Equations group solved the six one-variable linear equations with more success than did the non-Hands-On Equations group. The Hands-On Equations group scored a 72% accuracy rate for solving the six equations whereas the non-Hands-On Equations group scored a 59% rate of success. Thus, higher GPAs and ACT scores did not ensure more success in solving simple linear equations in this study.

6. Lower-level learners recognized that hands-on methods of teaching and learning are advantageous to them. In this study, 50% of the Hands-On Equations students identified themselves as “hands-on” learners. Of these 5 students, 4 took the ACT; each of their mathematics scores was either a 16 or 17. Also these same students were the ones who recalled emotional reactions rather than academic reactions to Hands-On Equations. The affective and hands-on aspects of learning may have a greater impact on lower-achieving students than on higher-achieving students. These findings are related to those of Witzel, Smith, and Brownell (2001) who advocated the use of the concrete-representation-abstract sequence to help students learn. Witzel et al. stated that students with learning disabilities require this three-phase support in learning abstract mathematical concepts. Kennedy (1986) concluded that middle

level students need manipulatives just as much as elementary students since the middle level concepts are just as abstract to that age student as elementary concepts are to younger children.

7. The amount and quality of recall of the higher-achieving students was notable. While unaware as sixth graders of the potential impact of learning algebra with the manipulative materials, Hands-On Equations, these students were the ones who later independently reflected and surmised that these materials had made a difference to them when they took a formal algebra class. Students 1 and 2 were the ones who had spontaneously reflected on the value of Hands-On Equations that day at their lockers during their senior year of high school. Student 1 had a GPA of 4.00 and an ACT mathematics score of 28; Student 2 was labeled as gifted and earned a GPA of 3.84 and an ACT mathematics score of 32.

These results are consistent with the research of others. One of the variables of long-term memory that were delineated by Semb, Ellis, and Araujo (1993) included individual differences. Howe (2000) concluded that slower learners forget more rapidly. Students who took higher-level courses in high school and earned better grades retained more (Terry, 2006).

8. The higher-achieving students verbalized the benefit of learning the structure of algebra early. They felt comfortable in their algebra classes and gave Hands-On Equations credit for instilling the algebraic structure that then allowed them to deal easily with new algebraic learning. Their observations are similar to Bruner's (1960) assertion when he used an algebra example to make his point that once a student grasps the fundamental ideas of equation solving, he or she can recognize that new equations are "not new at all, but only variants on a familiar theme" (p. 8). A key point made by Bruner was that early learning can make later learning more powerful and precise. He began with "the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). The fact that lower- as well as higher-achieving students from this study and the 1997 study were able to learn algebra at an early age is an example of Bruner's point. These results are similar to those found by others (Barclay, 1992; Busta, 1993; Leinenbach & Raymond, 1996).

9. For some of the students in this study, Hands-On Equations provided mental images that supported future learning of abstract algebra. Similar to the pilot study student's responses, Student 2 commented on the mental images from

Hands-On Equations that he used when formally learning algebra. These mental images are similar to Bruner's (1966) descriptions when he stated that in going from the concrete to the abstract, the learner retains and draws upon the "store of concrete images that served to exemplify the abstractions" (p. 65). Others (Bell & Tuley, 2006; Kennedy, 1986; Moyer, 2001) also defended the importance of manipulative materials to create concrete mental images that aid in the learning process.

10. Higher-achieving students in this study had memories that were more academic whereas the lower-achieving students had memories that were more emotional. The 6 students who had memories that were termed academic had a mean GPA of 3.25 and a mean ACT score of 22. The 3 students who had memories that were termed emotional had a mean GPA of 2.68. One of these 3 students did not take the ACT test. The 2 remaining students had a mean ACT score of 16.5. This conclusion is related to the research of Siegler (1998) who asserted that problems in learning mathematics have been attributed to aspects of three components: limited background knowledge, limited processing capacity, and limited conceptual understanding. In this study, the students with the lower GPAs and ACT scores had limited mathematical backgrounds and may

have had limited processing capacities. This may have accounted for the fact that their memories were not as academic as the higher performing students.

Recommendations

The following recommendations may be of interest to mathematics educators at all levels from upper elementary school through college and to researchers in this field.

1. Hands-On Equations should be taught to students in the middle level (sixth, seventh, or eighth grades) prior to their first formal algebra class. The results of the current study indicate the potential benefits for students who study Hands-On Equations. This study suggested that those benefits include access to foundational algebraic knowledge that could help students when they get to their first formal algebra class, alignment with visual or hands-on learning styles, and the promotion of student interest in mathematics. This recommendation is related to the one from Busta when she said that teachers should "utilize concrete manipulatives in instruction prior to ninth grade algebra" (p. 118). The results from this study confirm the information from other studies (Barclay, 1992; Busta, 1993; Leinenbach & Raymond, 1996) that suggest that Hands-On Equations is a viable vehicle in assisting

middle level students to learn algebra before they enroll in their first formal algebra class.

Bruner's (1960) work supports this recommendation. Bruner asserted that any subject could be taught effectively to any child in any stage of development. All students in the sixth-grade classroom where Hands-On Equations were taught in 1997 were included in the lessons. The recommendation that students learn algebra at an earlier age is supported by Siegler's (1998) claim that there is no single age at which students acquire a particular concept. Siegler's explanation that existing knowledge impacts a student's ability to learn new information supports the idea of learning algebra with Hands-On Equations prior to enrolling in a more formal algebra class.

2. Teachers of Hands-On Equations should be licensed or certified to teach middle level mathematics. Attending a one-day training session offered by Borenson and Associates would also be beneficial because the training allows a teacher to visualize how these materials could be utilized in a classroom. The Hands-On Equations lessons conducted in 1997 were taught by a teacher licensed to teach mathematics at the elementary school, junior high school, and high school levels, and who had attended a Borenson and Associates training day. Other teachers

of these materials may or may not have an interest in or understanding of the mathematical concepts inherent in these materials. Teachers without the necessary mathematical background and training may not be able to adequately teach Hands-On Equations.

An alternative suggestion is that teachers of Hands-On Equations be extensively trained by someone with the expertise and background knowledge to help the teachers both understand the algebraic concepts and how to teach them to students. This training could include but should not be limited to the one-day training session offered by Borenson and Associates.

3. A study that combines components of the 1997 study and this current study should be conducted with other classes of sixth-grade students. Teachers who are licensed to teach mathematics at the middle school level and have been trained in how to teach Hands-On Equations would teach at least the first two levels of Hands-On Equations to sixth graders. These students and comparable students who were not taught algebra with Hands-On Equations in the sixth grade would be interviewed as soon as possible after their first formal algebra class in either eighth or ninth grade. While ACT data would not be available at that time, student grades and attitudes might be

more easily gathered from larger numbers of students with this altered time schedule. Such a study could span a period of 3-4 years rather than 8. It is also recommended that this study be conducted with a diverse student population.

4. Another recommended study would investigate when students understand algebra during their first formal course. This proposed study would compare Hands-On Equations students with non-Hands-On Equations students. In the present study, the data from two students suggested that such a study could have interesting results. Student 12 understood algebra halfway through her first course whereas Student 6 remembered understanding algebra a few weeks into that first course. Student 12, a non-Hands-On Equations student, had a 31 ACT mathematics score as compared to Student 6, a Hands-On Equations student, who earned an ACT mathematics score of 16.

5. There should be more research on Hands-On Equations when all three levels of the system are taught. This recommendation is similar to one offered by Busta (1993) who recommended conducting a follow-up qualitative study where all three levels of Hands-On Equations were taught. Busta suggested conducting "a study of students in the middle grades over a 21 week period" (p. 118) as opposed to the seven weeks

and seven lessons that were used in her study. The current study involved both aspects of her recommendation, a qualitative study and all three levels of the program.

More use of these materials with accompanying research on the outcomes of such use is strongly recommended. Including the current study, only five studies are available that deal with Hands-On Equations. The current study found that students valued and enjoyed Hands-On Equations. These students concluded that Hands-On Equations made their first formal algebra class easier and more understandable than it seemed to be for their non-Hands-On Equations peers. In this study, the Hands-On Equations students liked mathematics more and were better able to solve one-variable linear equations when compared to the non-Hands-On Equations students. Should further research confirm the advantages found in this study, more teachers might use Hands-On Equations to help more students learn more algebra with less pain.

References

Agency for Instructional Technology. (2003). *Interview with Dr. Henry Borenson.* Retrieved October 6, 2003, from http://www.ait.net/workshops/borenson/

Alaska Department of Education & Early Development. (n.d.). *Performance standards for Alaska students.* Retrieved March 17, 2004, from http://www.eed.state.ak.us

Allen, R. (2003, Fall). Embracing math: Attitudes and teaching practices are changing—slowly. *Curriculum Update.* Alexandria, VA: Association for Supervision and Curriculum Development.

Arizona Department of Education. (2003). *Arizona academic content standards: Mathematics standards articulated by grade level.* Retrieved March 17, 2004, from http://www.ade.state.az.us

Bahrick, H. (1984). Semantic memory content in permastore: Fifty years of memory for Spanish learned in school. *Journal of Experimental Psychology: General, 113,* 1-29.

Bahrick, H., Bahrick, P., & Wittlinger, R. (1975). Fifty years of memory for names and faces: A cross-sectional approach. *Journal of Experimental Psychology: General, 104,* 54-75.

Bahrick, H., & Hall, L. (1993). Long intervals between tests can yield hypermnesia: Comments on Wheeler and Roediger. *Psychological Science, 4,* 206-207.

Baldwin High School. (1998). *Baldwin High School 1998-1999 course descriptions.* (Available from Baldwin High School, 415 Eisenhower, Baldwin City, KS 66006).

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory.* Englewood Cliffs, NJ: Prentice Hall.

Barclay, J. (1992). *A study of a manipulative approach to teaching linear equations to sixth grade students.* Unpublished master's thesis, Texas Woman's University, Denton.

Bartlett, F. (1932). *Remembering: A study in experimental and social psychology*. London: Cambridge University Press.

Battista, M. (1999). The mathematical miseducation of America's youth. *Phi Delta Kappan, 80*, 425-433.

Behr, M., Erlwanger, S., & Nichols, E. (1976). How children view equality sentences (PMCD Technical Report No. 3). Tallahassee: Florida State University. (ERIC Document Reproduction Service No. ED 144 802)

Bell, N., & Tuley, K. (2006). *Imagery: The sensory-cognitive connection for math.* Retrieved March 8, 2006 from http://www.ldonline.org/ld_indepth/math_skills/math_bell.html

Borenson and Associates. (n.d.a). *Brochure.* Retrieved April 23, 2005, from http://borenson.com

Borenson and Associates. (n.d.b). *Program validation.* Retrieved May 15, 2005, from http://borenson.com

Borenson and Associates. (n.d.c). *Sample problem.* Retrieved April 23, 2005, from http://borenson.com

Borenson, H. (1994). *The hands-on equations learning system.* Allentown, PA: Borenson and Associates.

Borget, M., & Gilroy, F. (1994). Interests and self-efficacy as predictors of mathematics/science-based career choice. *Psychological Reports, 75,* 753-754.

Brizuela, B., & Schliemann, A. (2003). Fourth graders solving equations. Manuscript in preparation.

Bruner, J. (1960). *The process of education.* New York: Random House.

Bruner, J. (1966). *Toward a theory of instruction.* Cambridge, MA: Harvard University Press.

Bruner, J. (1973). *Beyond the information given.* (J. Anglin, Ed.). New York: W.W. Norton & Company.

Busta, M. (1993). The relationship between middle school students' knowledge of the concept of variable and the use of concrete manipulatives. *Dissertation Abstracts International, 55* (03), 497, (UMI No. 9420286)

Carifio, J., & Nasser, R. (1994, April). *Algebra word problems: A review of the theoretical models and related research literature.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC Document Reproduction Service No. ED373965)

Carnopis, M. (1987, March 7). Math teacher accomplishes difficult feat: Made algebra fun, fourth-graders say. *Arkansas Democrat* (Available from Borenson and Associates, P.O. Box 3328, Allentown, PA 18106).

Carraher, D., Schliemann, A., Brizuela, B., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education, 37,* 87-115.

Center for Educational Testing and Evaluation, (n.d.a). *Kansas Mathematics Assessment – Spring 2000 Instructional Planning Report.* Retrieved May 9, 2006, from http://www.cete.ku.edu

Center for Educational Testing and Evaluation, (n.d.b). *Kansas Mathematics Assessment – Spring 2005 Instructional Planning Report.* Retrieved May 9, 2006, from http://www.cete.ku.edu

Cohen, J. (1992). A power primer. *Psychological Bulletin, 112,* 155-159.

Conway, M., Cohen, G., & Stanhope, N. (1991). On the very long-term retention of knowledge acquired through formal education: Twelve years of cognitive psychology. *Journal of Experimental Psychology: General, 120,* 395-409.

Costello, R. (Ed.). (1993). *The American college dictionary.* (3rd ed.). Boston: Houghton Mifflin Company.

Creswell, J. (1994). *Qualitative & quantitative approaches.* Thousand Oaks, CA: Sage Publications, Inc.

Dvorak, M. (1988). *Product review.* Allentown, PA: Borenson and Associates.

Educational Testing Service and the College Board. (1990). *Algebridge: Concept based instructional assessment.* Dedham, MA: Janson Publications.

Eisner, E. (1999). The uses and limits of performance assessment. *Phi Delta Kappan, 80*, 658-660.

Erlwanger, S., & Berlanger, M. (1983). Interpretations of the equal sign among elementary school children. In J. C. Bergeron & N. Herscovics (Eds.), *Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (vol. 1, pp. 250-258). Montreal: University of Montreal. (ERIC Document Reproduction Service No. ED 289 688)

Esty, W. (1992, Fall). Language concepts of mathematics. *Focus on Learning Problems in Mathematics, 14,* 31-54.

Esty, W. (1999). *The language of mathematics* (16th ed.). Bozeman, MT: Author.

Esty, W. (2000, April). The language of mathematics. In *The language of mathematics: A language you should teach.* Symposium conducted at the meeting of the Kansas Section of the Mathematical Association of America, Baldwin City, Kansas.

Esty, W., & Teppo, A. (1994, Winter). A general-education course emphasizing mathematical language and reasoning. *Focus on Learning Problems in Mathematics, 16,* 13-35.

Faulkner, K., Levi, L., & Carpenter, T.P. (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics*, *6*, 232-236.

Fennema, E., & Franke, M. (1992). Teachers' knowledge and its impact. In D.Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan Publishing Company.

Fenwick, P. (n.d.). *Retrospective studies on NDEs.* Retrieved July 5, 2006, from http://iands.org/research/fenwick1.php

Flavell, J. (1992). Cognitive development: Past, present, and future. *Developmental Psychology, 28(6)*, 998-1005.

Gall, M., Gall, J., & Borg, W. (2003). *Educational research: An introduction.* Boston: Allyn and Bacon.

Ghazi, P. (2000, October 24). Catch them young: Fear + loathing = algebra. Unless you're one of the thousands of 9-year-old Americans to have discovered that algebra = fun. *The Guardian.* Retrieved October 6, 2003, from http://education.guardian.co.uk/old/schools/story/0,10044,386716,00.html

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 1-23). Reston, VA: National Council of Teachers of Mathematics.

Greenes, C., & Findell, C. (1999). Developing students' algebraic reasoning abilities. In L. Stiff & F. Curcio (Eds.), *Developing mathematical reasoning in grades K-12* (pp. 127-137). Reston, VA: National Council of Teachers of Mathematics.

Hackett, G., & Betz, N.E. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. *Journal for Research in Mathematics Education, 20,* 261-273.

Hartshorn, R., & Boren, S. (1990). Experiential learning of mathematics: Using manipulatives. *ERIC Digest.* Retrieved May 8, 2005, from www.eric.ed.gov

Hatfield, M., Edwards, N., Bitter, G., & Morrow, J. (2005). *Mathematics methods for elementary and middle school teachers.* Hoboken, NJ: John Wiley & Sons, Inc.

Herbert, E. (1985). Manipulatives are good mathematics. *Arithmetic Teacher, 32*, 4.

Herbert, K., & Brown, R. (1997). Patterns as tools for algebraic reasoning. *Teaching Children Mathematics*, *3,* 340-344.

Hohn, R. (1995). *Classroom learning & teaching.* White Plains, NY: Longman Publishers USA.

Howard, G. (1980). Response-shift bias: A problem in evaluating interventions with pre-post self-reports. *Evaluation Review, 4,* 93-106.

Howe, M. (2000). *The fate of early memories: Developmental science and the retention of childhood experiences.* Washington, DC: American Psychological Association.

International Study Center. (2005). *Third International Mathematics and Science Study-1995.* Retrieved May 30, 2005, from http://timss.bc.edu/timss1995.html

Jarratt, S., Mack, K., & Watson, S. (n.d.) *Retrospective writing histories.* Retrieved July 5, 2006, from http://www.writing.uci.edu/retrowriting.pdf

Johnston, E. (2001). The repeated reproduction of Bartlett's *Remembering*. *History of Psychology, 4,* 341-366.

Kansas State Board of Education. (2004). *Kansas curricular standards for mathematics.* Topeka, KS: Author.

Kansas State Department of Education. (n.d.). *Building student mastery of algebraic concepts by race and gender.* Retrieved February 1, 2006 from http://www.ksde.org

Kansas State Department of Education. (2001). *Released items from the 2001 fourth grade Kansas mathematics assessment.* Retrieved May 29, 2005, from http://www.ksde.org/outcomes/mathexamples.html

Kennedy, L. (1986). A rationale. *Arithmetic Teacher, 33.* 6-7.

Kenney, P., & Silver, E. (1997). Probing the foundations of algebra: Grade-4 pattern items in NAEP. *Teaching Children Mathematics*, *3,* 268-274.

Kenney-Benson, G., Pomerantz, E., Ryan, A., & Patrick, H. (2006). Sex differences in math performance: The role of children's approach to schoolwork. *Developmental Psychology, 42,* 11-26.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, *12*, 317-326.

Kieran, C. (1989). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 33-56). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan Publishing Company.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Lane, N. (1996). *On release of the curriculum analysis aspect of the Third International Mathematics and Science Study.* Retrieved June 5, 2003, from http://ustimss.msu.edu/nsfstmnt.htm

Lawson, D. (1990). The problem, the issues that speak to change. In E. Edwards (Ed.), *Algebra for everyone* (pp. 1-6). Reston, VA: National Council of Teachers of Mathematics.

Leinenbach, M., & Raymond, A. (1996). *A two-year collaborative action research study on the effects of a "hands-on" approach to learning algebra* (Report No. SE058808). East Lansing, MI: National Center for Research on Teacher Learning. (ERIC Document Reproduction Service No. ED398081)

Lemlech, J. (2002). *Curriculum and instructional methods for the elementary and middle school.* Upper Saddle River, NJ: Merrill Prentice Hall.

Lengfelder, A. & Heller, K. (n.d.) *German olympiad studies: Findings from a retrospective evaluation and from in-depth interviews. Where have all the gifted females gone?* Retrieved June 5, 2006 from http://olympiadprojects.com/v2/pubs_web%5Cch4_SS.pdf

Lent, R., Brown, S., & Gore, P. (1997). Discriminant and predictive validity of academic self-concept, academic self-efficacy, and mathematics-specific self-efficacy. *Journal of Counseling Psychology, 44*, 307-315.

Lent, R., Lopez, F., & Bieschke, K. (1991). Mathematics self-efficacy: Sources and relation to science-based career choice. *Journal of Counseling Psychology, 38,* 424-430.

Locke, L., Spirduso, W., & Silverman, S. (2000). *Proposals that work.* Thousand Oaks, CA: Sage Publications, Inc.

Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education, 28,* 26-47.

MacGregor, M., & Stacey, K. (1999). A flying start to algebra. *Teaching Children Mathematics*, *6*, 78-85.

Mager, R. (1968). *Developing attitude toward learning.* Palo Alto, CA: Fearon Publishers.

Marsh, H., Dowson, M., Pietsch, J., & Walker, R. (2004). Why multicollinearity matters: A reexamination of relations between self-efficacy, self-concept, and achievement. *Journal of Educational Psychology, 96,* 518-522.

Maxwell, J. (1996). *Qualitative research design: An interactive approach.* Thousand Oaks, CA: Sage Publications, Inc.

Maxwell, S., & Delaney, H. (2000). *Designing experiments and analyzing data: A model comparison perspective.* Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Mayer, R. (1999). *The promise of educational psychology.* Upper Saddle River, NJ: Prentice Hall.

Merriam, S. (1998). *Qualitative research and case study applications in education.* San Francisco: Jossey-Bass Publishers.

Moyer, P. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47(2)*, 175-197.

Moyer, P., & Jones, G. (1998). Tools for cognition: Student free access to manipulative materials in control- versus autonomy-oriented middle grades teachers' classrooms. *Clearinghouse for*

Science, Mathematics, and Environmental Education. (ERIC Document Reproduction No. ED 420 524)

National Center for Education Statistics. (n.d.). *NAEP questions.* Retrieved August 7, 2004, from http://nces.ed.gov/nationsreportcard/itmrls

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1992). *Algebra for the twenty-first century: Proceedings of the August 1992 conference.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

Neale, D. (1969). The role of attitudes in learning mathematics. *The Arithmetic Teacher, 8,* 631-640.

Pajares, F., & Miller, M. (1995). Mathematics self-efficacy and mathematics performances: The need for specificity of assessment. *Journal of Counseling Psychology, 42,* 190-198.

Patton, M. (2002). *Qualitative research & evaluation methods.* Thousand Oaks, CA: Sage Publications, Inc.

Phi Delta Kappa. (2000). *A Phi Delta Kappa professional development institute, making algebra child's play!: A hands-on equations workshop* [Advertisement]. Bloomington, IN: Author.

Pietsch, J., Walker, R., & Chapman, E. (2003). The relationship among self-concept, self-efficacy, and performance in mathematics during secondary school. *Journal of Educational Psychology, 95,* 589-603.

Resnick, L. (1989). Developing mathematical knowledge. *American Psychologist, 44,* 162-169.

Rippey, R., Geller, L., & King, D. (1978). Retrospective pretesting in the cognitive domain. *Evaluation Quarterly, 2,* 481-491.

Rittle-Johnson, B., & Alibali, M. (1999). Conceptual and procedural knowledge of mathematics : Does one lead to the other? *Journal of Educational Psychology*, *91*, 175-189.

Rubenstein, R., & Thompson, D. (2001). Learning mathematical symbolism: Challenges and instructional strategies. *Mathematics Teacher*, *94*, 265-271.

Rubin, H., & Rubin, I. (1995). *Qualitative interviewing: The art of hearing data.* Thousands Oaks, CA: Sage Publications, Inc.

Sa'ar, R. (n.d.). *An innovation to improve children's math scores.* Retrieved August 22, 2002, from www.haaretzdaily.com

Saenz-Ludlow, A., & Walgamuth, C. (1998). Third graders' interpretation of equality and the equal symbol. *Educational Studies in Mathematics*, *35*, 153-187.

Saul, M. (2001). Algebra: What are we teaching? In A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 35-43). Reston, VA: National Council of Teachers of Mathematics.

Sax, G. (1997). *Principles of educational and psychological measurement and evaluation.* Belmont, CA: Wadsworth Publishing Company.

Schmidt, W. (1996, November 27). U.S. math, science education in need of vision. *USA Today, 11/27/96.* Retrieved October 10, 2004, from http://ustimss.msu.edu/editoria.htm

Schmidt, W. (2000). *Paying the price of "no change."* Retrieved October 10, 2004, from http://ustimss.msu.edu/editoria.htm

Semb, G., Ellis, J., & Araujo, J. (1993). Long-term memory for knowledge learned in school. *Journal of Educational Psychology, 85,* 305-316.

Siegler, R. (1998). *Children's thinking.* Upper Saddle River, NJ: Prentice Hall.

Siegler, R. (2003). Implications of cognitive science research for mathematics education. In J. Kilpatrick, W.G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 289-303). Reston, VA: National Council of Teachers of Mathematics, Inc.

Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research, 95,* 323-332.

Stewart, M. (2003). From tangerines to algorithms. *Instructor, 112,* 20-23.

Stigler, J., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). *The TIMSS Videotape Classroom Study: Methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States.* Washington, DC: National Center for Education Statistics. Retrieved October 24, 2004, from http://nces.ed.gov/timss

Suh, J. (2005). Third graders' mathematics achievement and representation preference using virtual and physical manipulatives for adding fractions and balancing equations. *Dissertation Abstracts International, 66*(02), 528.

Suter, L. & Frechtling, J. (2000). *Guiding principles for mathematics and science education research methods: Report of a workshop.* Retrieved July 5, 2006, from http://www.nsf.gov/pubs/2000/nsf00113/nsf00113.html

Terry, W. (2006). *Learning and memory: Basic principles, processes, and procedures.* Boston: Pearson Education, Inc.

Tierney, C., & Nemirovsky, R. (1997). Mathematics of change: Ins and outs or ups and downs. *Teaching Children Mathematics, 3,* 336-345.

Tulving, E. (1985). How many memory systems are there? *American Psychologist, 40,* 385-398.

U.S. National Research Center. (1996). *Summary of findings.* Retrieved October 10, 2004, from http://ustimss.msu.edu/summary.htm

U.S. National Research Center. (1997). *What can we learn from the U.S. 4th grade achievement results?* Retrieved October 10, 2004, from http://ustimss.msu.edu/backgroundreport.html

Usiskin, Z. (1992). Where does algebra begin? Where does algebra end? In *Algebra for the twenty-first century: Proceedings of the August 1992 conference* (pp. 27-28). Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (1996). Mathematics as a language. In P. C. Elliott & M. J. Kenney (Eds.), *Communication in mathematics, K-12 and beyond* (pp. 231-243). Reston, VA: National Council of Teachers of Mathematics.

Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning.* London: Falmer Press.

Von Rotz, L., & Burns, M. (2002). *Grades K-2 lessons for algebraic thinking*. Sausalito, CA: Math Solutions Publications.

Wadsworth, B. (1984). *Piaget's theory of cognitive and affective development.* New York: Longman.

Watson, J. (1925). *Behaviorism.* New York: Norton.

Weiss, D. (2006). Keeping it real: The rationale for using manipulatives in the middle grades. *Mathematics Teaching in the Middle School, 11(5)*, 238-242.

Witzel, B., Smith, S., & Brownell, M. (2001). How can I help students with learning disabilities in algebra? *Intervention in School and Clinic, 37(2)*, 101-104.

Wolcott, H. (1990). *Writing up qualitative research.* Thousand Oaks, CA: Sage Publications, Inc.

Wolcott, H. (1994). *Transforming qualitative data: Description, analysis, and interpretation.* Thousand Oaks, CA: Sage Publications, Inc.

Woolf, H. (Ed.) (1981). *Webster's new collegiate dictionary.* Springfield, MA: G. & C. Merriam Company.

APPENDIX A

SUMMARY OF LESSON OBJECTIVES

FOR HANDS-ON EQUATIONS

Summary of Lesson Objectives for Hands-On Equations

Note: TLW stands for "The learner will."

Level I

1 TLW use a balance scale to grasp the concept that each side of the equal sign needs to be equal.

2 TLW use the student manipulatives to intuitively learn about equations, variables, and unknowns.
TLW guess and check to solve equations.
TLW label the blue pawn as "x."

3 TLW use the first "legal move" to subtract the blue pawn.

4 TLW subtract the same number cube value.

5 TLW take away pawns as part of the setup process. (Subtraction is in the original equation.)

6 TLW solve equations involving parentheses.

7 TLW transfer concrete experiences to a pictorial system involving only pencil and paper.

Level II

8 TLW manipulate the white pawn using the same legal moves from Level I.

9 TLW recognize the blue pawn and the white pawn as opposites. TLW add x and * to get zero. x + * = 0.
TLW distinguish between equations and expressions.
TLW evaluate expressions involving x and * with the value of x or * given.

10 TLW remove a blue pawn and a white pawn if they appear on the same side of the setup. (This is a new legal move called "removing a zero value.")

11 TLW add a zero value.
TLW isolate the number cube as a strategy by eliminating the pawn next to the number cube.

12 TLW create a zero by adding a blue pawn to each side of the setup.

13 TLW work with the problem in order to have number cubes on only one side of the setup.

14 TLW add a "convenient zero" in order to complete the setup.

15 TLW recognize a written "(-x)" as *, the white pawn.

16 TLW transfer concrete experiences to a pictorial system.

Level III

17 TLW add and subtract positive and negative integers using the red and green cubes.

18 TLW work with equations involving the green cube.

19 TLW add a convenient zero made up of number cubes.

20 TLW solve equations using all manipulatives with pawns of only one color in an equation.

21 TLW solve equations using all manipulatives.

22 TLW solve equations containing multiples of parenthetical expressions involving the green cube. Example: 2(x - 1)

23 TLW subtract a multiple of a parenthetical expression.

24 TLW solve equations that use "(-x)."

25 TLW transfer concrete experiences to a pictorial system.

26 Optional lesson: TLW transition from the pictorial system to the traditional written notation.

Note: TLW stands for "The learner will."

APPENDIX B

SAMPLE PAGES FROM HANDS-ON EQUATIONS

TEACHER MANUAL

Since algebra is basic to the further study of mathematics and science, students who do not gain access to algebra will find the doors closed to many professional career opportunities. Traditionally, there has been a chasm in mathematics education, in that students have been expected to grasp the concepts, symbols, and methods of algebra at the abstract level in the 8th or 9th grade. *An intuitive, concrete and pictorial foundation for algebra has been missing in mathematics education!*

The *Hands-On Equations* patented teaching methodology provides this entranceway to algebra. *This methodology enables students from grades 3 to adult to meaningfully gain access to, and advance in, the world of basic algebra.*

Early success with algebraic concepts provides students with a tremendous sense of mathematical power and self-confidence, bolsters students' mathematical interests, and provides students with an intuitive and concrete foundation for later algebraic work.

Below follows a description of *The Hands-On Equations Learning System*, Level I. This sequence of lessons, with students using the concrete materials of the program, has been found successful with students from ages 8 to 84.

The Hands-On Equations® Learning System
Level I

Objective: By the end of the seventh lesson, elementary school students (third grade and up) will be able to physically set up and solve such equations as

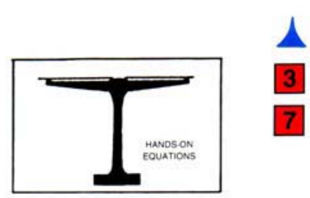
$$2x + x + x + 2 = 2x + 10$$

and

$$2(x + 4) + x = x + 16.$$

Materials Needed Per Student:

Eight blue pawns
Two red cubes, numbered 0-5
Two red cubes, numbered 5-10
A laminated balance scale



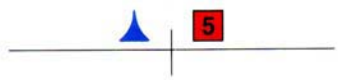
<u>Materials Needed by the Teacher</u>:

Pawns and cubes as above, but larger;

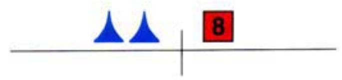
A stationary physical scale

Lesson #1

In the first lesson, the teacher displays on the physical scale in front of the classroom, problems such as

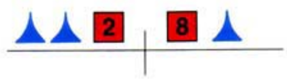


and



FOR HOME USE:
Please display these problems on the enclosed laminated balance scale.

Once students grasp the concept that both sides of the scale must have the same value for the scale to balance, they see that the pawn in the first problem is worth 5, and that in the second problem it is worth 4. Students can then be presented with other "physical equations" which they are to solve by trial and error methods.

Ex. 

The students see that "1" does not work since both sides are not equal. "2" does not work, etc. "6" does work since the left side is now 14 and so is the right side. The students are informed that the pawn has a special name, "x," and that there is a special way of writing the answer:

$$x = 6, \text{ check: } 14 \stackrel{\checkmark}{=} 14.$$

The students are given Student Kits so that they can set up the worksheet problems at their desks. (On the student setup, it is helpful if the number-cubes are facing <u>upward</u>, i.e., facing the <u>ceiling</u>, so that the teacher can easily see if the students have the correct setup.)

Other examples which the teacher can assign in this lesson include

$$2x + x - x + 1 = x + 9$$

and

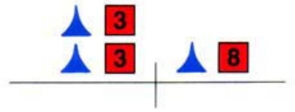
$$4 + 3x - 2x + x = x + 5.$$

Lesson #6

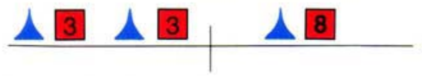
In this lesson, students learn to solve such equations as

$$2(x + 3) = x + 8.$$

They learn that the "2" outside the parenthesis means that what is inside the parenthesis, the "$x + 3$," is to be doubled. Hence, the student setup for this problem is



By having the students display the doubled portion in two rows on the mat, the teacher can easily check that the correct elements have been doubled. (In the teacher setup, the pawns and cubes are placed <u>next</u> to each other,



so that they are visible to the class.) By subtracting one x from each side, we see that $x = 2$; check $10 \stackrel{\checkmark}{=} 10$.

Other examples the teacher can give in this lesson include

$$2(2x + 1) = 18$$

and

$$2(x + 4) + x = x + 16.$$

Comments on Lesson #6

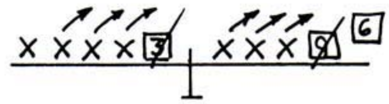
It is fascinating to see that elementary school students, when instructed in this manner, have little difficulty in working with a multiple of a parenthetical expression. Occasionally, they "discover" the distributive law on their own and double each element inside the parenthesis in sequence.

Lesson #7

In this lesson, students transfer their concrete, hands-on experience in solving algebraic linear equations to a pictorial system involving only pencil and paper. The technique is illustrated in the two examples below.

Ex. $4x + 3 = 3x + 9$

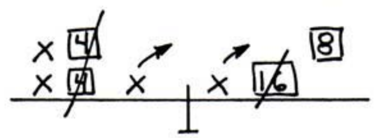
Solution:



So, $x = 6$. Check: $27 \overset{\checkmark}{=} 27$.

Ex. $2(x + 4) + x = x + 16$

Solution:



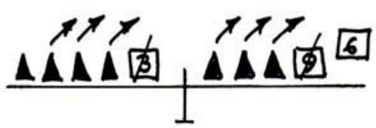
So, $x = 4$. Check: $20 \overset{\checkmark}{=} 20$.

Thus in this pictorial notation, the student draws a scale, and on it, places written "x's" (instead of pawns), places written boxed-numerals (instead of number-cubes), and crosses off or places arrows above anything that is to be taken away. Once the answer is obtained, the student goes back to the initial pictorial setup *(redrawn, for clarity)* in order to carry out the check.

Note: Some students prefer to use pictures of the pawns, rather than written "x's," to represent the physical pawns.

Ex. $4x + 3 = 3x + 9$

Solution:



So, $x = 6$. Check: $27 \overset{\checkmark}{=} 27$.

This pictorial notation more closely resembles the actual physical setup and is therefore easier for some younger students and some students with learning disabilities to understand. This notation is perfectly acceptable. (The picture of the blue pawn is shaded in, to distinguish it, in Level II, from the picture of the white pawn which is not shaded in.)

APPENDIX C

1997 PRETEST AND POSTTEST WITH ANSWER KEY

Name________________________________ Date______________________

Do your work on the blank paper you are given. Then put the answer on the line under each problem.

1. Three times a number, increased by 1, is 25. Find the number.

1. __________

2. You can buy 5 small pizzas for the same price as 3 small pizzas and 10 one dollar drinks. How much does each pizza cost?

2. __________

3. Brian buys 1 pack of baseball cards to add to the 2 cards a friend gave him. Then his mother gives him 2 more packs as a special treat. Now he has as many cards as Marcus who owns 1 pack plus 12 loose cards. How many cards are in each pack?

3.__________

4. Four times a number, increased by 3, is the same as twice the number, increased by 9. Find the number.

4. __________

5. Erin can buy 5 putt-putt tickets and 2 one dollar boxes of popcorn for the same price as 3 putt-putt tickets and 12 one dollar boxes of popcorn. How much does each putt-putt ticket cost?

5. __________

6. John is 6 years older than Kathy. Together, their ages equal four times Kathy's age. How old is each?

6. __________

7. Allison has 2 aquariums. In each aquarium she has 2 families of guppies and 3 tetras. Leigh has 1 aquarium with 10 tetras and 3 families of guppies. Allison and Leigh have the same number of fish and their guppy families each have the same number of members. How many guppies are in each family?

7. __________

1997 Pretest and Posttest Answer Key

1) $3x + 1 = 25$
   $3x = 24$
   $x = 8$ — The number is 8.

2) $5x = 3x + 10$
   $2x = 10$
   $x = 5$ — Each pizza costs $5.00.

3) $x + 2 + 2x = x + 12$
   $3x + 2 = x + 12$
   $2x = 10$
   $x = 5$ — There are 5 cards in each pack.

4) $4x + 3 = 2x + 9$
   $2x = 6$
   $x = 3$ — The number is 3.

5) $5x + 2 = 3x + 12$
   $2x = 10$
   $x = 5$ — Each putt-putt ticket costs $5.00.

6) $x + (x + 6) = 4x$
   $2x + 6 = 4x$
   $6 = 2x$
   $3 = x$ — Kathy is 3 years old and John is 9 years old.

7) $2(2x + 3) = 3x + 10$
   $4x + 6 = 3x + 10$
   $x = 4$ — There are 4 guppies in each family.

APPENDIX D

1997 DATA RESULTS

Student	Pre	Post	Difference
1	1	5	4
2	3	5	2
3	2	3	1
4	1	4	3
5	1	4	3
6	2	5	3
7	1	3	2
8	3	4	1
9	1	2	1
10	4	5	1
11	2	4	2
12	1	5	4
13	1	5	4
14	3	7	4
15	5	7	2
16	3	7	4
17	3	6	3
18	6	6	0

Difference	
Mean	2.444444444
Standard Error	0.304886374
Median	2.5
Mode	4
Standard Deviation	1.293523334
Sample Variance	1.673202614
Kurtosis	-1.129476929
Skewness	-0.229508809
Range	4
Minimum	0
Maximum	4
Sum	44
Count	18
Confidence Level(95.0%)	0.643254919

Pre		*Post*	
Mean	2.388888889	Mean	4.833333333
Standard Error	0.353681745	Standard Error	0.335775368
Median	2	Median	5
Mode	1	Mode	5
Standard Deviation	1.500544563	Standard Deviation	1.42457424
Sample Variance	2.251633987	Sample Variance	2.029411765
Kurtosis	0.522246119	Kurtosis	-0.380634321
Skewness	1.004473263	Skewness	-0.083930492
Range	5	Range	5
Minimum	1	Minimum	2
Maximum	6	Maximum	7
Sum	43	Sum	87
Count	18	Count	18
Confidence Level(95.0%)	0.746204298	Confidence Level(95.0%)	0.708425092

APPENDIX E

1997 CONTENT VALIDITY BY EXPERTS

Background information:

A seven-question, word-problem test was compiled from Hands-On Equations Verbal Problems Sides A and B (Borenson, 1994) to measure sixth-grade student achievement and to use as both the pretest and the posttest. That test is printed below. Problem solving was the purpose of the pretest and posttest; all seven test items were word problems to solve.

Statement to validate:

I have examined the test printed below. In my opinion, the seven questions directly relate to student ability to solve problems that are algebraic by design. I have a Ph.D. in mathematics and have taught algebra courses.

R. J. Fraga   12 May 2005

Robert Fraga, Ph.D.   Date

1. Three times a number, increased by 1, is 25. Find the number.
2. You can buy 5 small pizzas for the same price as 3 small pizzas and 10 one dollar drinks. How much does each pizza cost?
3. Brian buys 1 pack of baseball cards to add to the 2 cards a friend gave him. Then his mother gives him 2 more packs as a special treat. Now he has as many cards as Marcus who owns 1 pack plus 12 loose cards. How many cards are in each pack?
4. Four times a number, increased by 3, is the same as twice the number, increased by 9. Find the number.
5. Erin can buy 5 putt-putt tickets and 2 one dollar boxes of popcorn for the same price as 3 putt-putt tickets and 12 one dollar boxes of popcorn. How much does each putt-putt ticket cost?
6. John is 6 years older than Kathy. Together, their ages equal four times Kathy's age. How old is each?
7. Allison has 2 aquariums. In each aquarium she has 2 families of guppies and 3 tetras. Leigh has 1 aquarium with 10 tetras and 3 families of guppies. Allison and Leigh have the same number of fish and their guppy families each have the same number of members. How many guppies are in each family?

Background information:

A seven-question, word-problem test was compiled from Hands-On Equations Verbal Problems Sides A and B (Borenson, 1994) to measure sixth-grade student achievement and to use as both the pretest and the posttest. That test is printed below. Problem solving was the purpose of the pretest and posttest; all seven test items were word problems to solve.

Statement to validate:

I have examined the test printed below. In my opinion, the seven questions directly relate to student ability to solve problems that are algebraic by design. I have a Ph.D. in mathematics and have taught algebra courses.

Jean T. Johnson 5/12/05

Jean Johnson, Ph.D. Date

1. Three times a number, increased by 1, is 25. Find the number.

2. You can buy 5 small pizzas for the same price as 3 small pizzas and 10 one dollar drinks. How much does each pizza cost?

3. Brian buys 1 pack of baseball cards to add to the 2 cards a friend gave him. Then his mother gives him 2 more packs as a special treat. Now he has as many cards as Marcus who owns 1 pack plus 12 loose cards. How many cards are in each pack?

4. Four times a number, increased by 3, is the same as twice the number, increased by 9. Find the number.

5. Erin can buy 5 putt-putt tickets and 2 one dollar boxes of popcorn for the same price as 3 putt-putt tickets and 12 one dollar boxes of popcorn. How much does each putt-putt ticket cost?

6. John is 6 years older than Kathy. Together, their ages equal four times Kathy's age. How old is each?

7. Allison has 2 aquariums. In each aquarium she has 2 families of guppies and 3 tetras. Leigh has 1 aquarium with 10 tetras and 3 families of guppies. Allison and Leigh have the same number of fish and their guppy families each have the same number of members. How many guppies are in each family?

Background information:

A seven-question, word-problem test was compiled from Hands-On Equations Verbal Problems Sides A and B (Borenson, 1994) to measure sixth-grade student achievement and to use as both the pretest and the posttest. That test is printed below. Problem solving was the purpose of the pretest and posttest; all seven test items were word problems to solve.

Statement to validate:

I have examined the test printed below. In my opinion, the seven questions directly relate to student ability to solve problems that are algebraic by design. I have a Ph.D. in mathematics and have taught algebra courses.

05/13/05

Mircea Martin, Ph.D. Date

1. Three times a number, increased by 1, is 25. Find the number.
2. You can buy 5 small pizzas for the same price as 3 small pizzas and 10 one dollar drinks. How much does each pizza cost?
3. Brian buys 1 pack of baseball cards to add to the 2 cards a friend gave him. Then his mother gives him 2 more packs as a special treat. Now he has as many cards as Marcus who owns 1 pack plus 12 loose cards. How many cards are in each pack?
4. Four times a number, increased by 3, is the same as twice the number, increased by 9. Find the number.
5. Erin can buy 5 putt-putt tickets and 2 one dollar boxes of popcorn for the same price as 3 putt-putt tickets and 12 one dollar boxes of popcorn. How much does each putt-putt ticket cost?
6. John is 6 years older than Kathy. Together, their ages equal four times Kathy's age. How old is each?
7. Allison has 2 aquariums. In each aquarium she has 2 families of guppies and 3 tetras. Leigh has 1 aquarium with 10 tetras and 3 families of guppies. Allison and Leigh have the same number of fish and their guppy families each have the same number of members. How many guppies are in each family?

APPENDIX F

INFORMED CONSENT STATEMENT

Approved by the Human Subjects Committee University of Kansas, Lawrence Campus (HSCL). Approval expires one year from 5/3/2005.

Informed Consent Statement

I am conducting interviews and gathering data in partial fulfillment of the requirements for a Ph.D. in mathematics education that I am pursuing at the University of Kansas. The Department of Teaching and Leadership at the University of Kansas supports the practice of protection for human subjects participating in research. The following information is provided for you to decide whether you wish to participate in the present study. Even if you agree to participate, you are free to withdraw at any time. In addition, you are free to not answer any of the questions I might ask you.

I have chosen to do a study that will provide insight into the perceived value of the *Hands-On Equations Learning System*. These materials may or may not help elementary and middle school students understand algebra before they get to more formal algebra classes but there have been few studies to document the effectiveness of the *Hands-On Equations Learning System*. The reactions of students who did and who did not experience these materials are important to this study.

Although you have tentatively agreed to participate in my study, your participation in this study is strictly voluntary. I do not anticipate that any of the questions I ask will cause you discomfort, but please feel free to not answer any questions you choose. I assure you that your name will not be associated in any way with the research findings. If you would like additional information concerning this study before, during, or after its completion, please feel free to contact me, Merrie Skaggs, by phone or e-mail. Information on my advisor Dr. Gay is also included.

Sincerely,

Merrie Skaggs, Principal Investigator
Assistant Professor
Baker University
Department of Education
205b Case Hall
Baldwin City, Kansas 66006
785-594-8491
mskaggs@bakeru.edu

Susan Gay, Faculty Advisor
Associate Professor
University of Kansas
Department of T & L, SOE
JRP, Room 341
Lawrence, Kansas 66045
785-864-9676
sgay@ku.edu

I agree to participate in this study and I understand that my interview and data I provide will be confidential. I am 18 years old or older.

Signature	Printed Name	Date

I also give my permission for Merrie Skaggs to view my high school transcript and collect my GPA, my mathematics grades, and my ACT scores with the understanding that all of this information will remain anonymous and a part of group data. I am 18 years old or older.

Signature	Printed Name	Date

With my signature I acknowledge that I have received a copy of this consent form to keep.

APPENDIX G

PROTOCOL QUESTIONS AND

SIX EQUATIONS TO SOLVE

Protocol Questions

1. Were you in Mrs. Roberts's sixth-grade class when Hands-On Equations were taught? ***(This question is for verification. I will already have this information. If the answer is "no," skip to Questions 4 through 11, 15 and 16.)*** **[1]**

2. Tell me all you can recall about Hands-On Equations. **[1]**

3. Are there any reactions to Hands-On Equations that you recall? **[1]**

4. Did you take algebra in junior high, high school, or college? ***(Note: The typical college-preparatory track for Baldwin students at that time was Algebra I – 9th grade, Algebra II – 10th grade, Geometry – 11th grade, and Advanced Math – 12th grade.)*** **[2, 4b]**

5. Describe your experiences when you got to your first algebra class. **[2, 4a]**

6. Would you describe algebra as easy or hard? Explain your answer. **[2, 3, 4a]**

7. You have already agreed to let me view your grades. Do you remember your grades in your math classes? Would you mind sharing those with me? ***(The purpose of this question is to prompt any additional memories of algebra and student reactions to the grades received.)*** **[4a, 4b]**

8. Do you remember your ACT score? What was your ACT score in math? ***(These questions may not be necessary; I will get this information.)*** **[4b]**

9. Here are 10 subject matter areas: art, health, language arts (English), math, music, physical education (PE), reading, science, social studies, and writing. **(*Have these written out for the students to view.)*** What rank would you assign to math if 1 indicates your favorite subject and 10 indicates your least favorite subject? **[4a]**

10. Why did you rank math the way you did? **[4a]**

11. Do you think of yourself as being "good at math" or "not so good"? **[3]**

12. Do you think that the Hands-On Equations experience made a difference in your learning math later on in high school or college? If so, how? If not, why not? **[2]**

13. Do you think other sixth graders should learn Hands-On Equations? Why or why not? **[1]**

14. Can you give me three facts you recall about Hands-On Equations? **[1]**

15. Tell me about two positive experiences in math that you recall. Tell me about two negative experiences in math that you recall. **[3, 4a]**

16. Would you please solve the following equations: ***(Have these written out.)*** **[4c]**

$2(x + 4) + x = x + 16$	$2x + (-x) + 3 = 2(-x) + 15$
$2x + x + x + 2 = 2x + 10$	$2x - 3(-x) = 20 + x$
$2(-x + 8) - (-3) = 2x + 3$	$2x - 2(-x + 4) = x + (-2)$

Note: The numbers in brackets [ ] indicate the research question(s) that this protocol question addresses.

Protocol Questions for Hands-On Equations Students

1. Were you in Mrs. Roberts's sixth-grade class when Hands-On Equations were taught?

2. Tell me all you can recall about Hands-On Equations.

3. Are there any reactions to Hands-On Equations that you recall?

4. Did you take algebra in junior high, high school, or college? ***(If "no," skip to 7.)***

5. Describe your experiences when you got to your first algebra class.
   Did you understand algebra in that first class? At what time did you understand it? What was a main idea of algebra, or what was the point?

6. Would you describe algebra as easy or hard? Explain your answer.

7. You have already agreed to let me view your grades. Do you remember your grades in your math classes? Would you mind sharing those with me?

8. Do you remember your ACT score? What was your ACT score in math?

9. Here are 10 subject matter areas: art, health, language arts (English), math, music, physical education (PE), reading, science, social studies, and writing. ***(Have these written out for the students to view.)*** What rank would you assign to math if 1 indicates your favorite subject and 10 indicates your least favorite subject?

10. Why did you rank math the way you did?

11. Do you think of yourself as being "good at math" or "not so good"? Why did you answer the way you did?

12. Do you think that the Hands-On Equations experience made a difference in your learning math later on in high school or college? If so, how? If not, why not?

13. Do you think other sixth graders should learn Hands-On Equations? Why or why not?

14. Can you give me three facts that you recall about Hands-On Equations?

15. Tell me about two positive experiences in math that you recall. Tell me about two negative experiences in math that you recall.

16. Would you please solve the following equations: ***(Have these written out.)***

$2(x + 4) + x = x + 16$	$2x + (-x) + 3 = 2(-x) + 15$
$2x + x + x + 2 = 2x + 10$	$2x - 3(-x) = 20 + x$
$2(-x + 8) - (-3) = 2x + 3$	$2x - 2(-x + 4) = x + (-2)$

Protocol Questions for Non-Hands-On Equations Students

1. Were you in Mrs. Roberts's sixth-grade class when Hands-On Equations were taught? ***(If "no," skip to 4.)***

2. Tell me all you can recall about Hands-On Equations.

3. Are there any reactions to Hands-On Equations that you recall?

4. Did you take algebra in junior high, high school, or college? ***(If "no," skip to 7.)***

5. Describe your experiences when you got to your first algebra class.
   Did you understand algebra in that first class? At what time did you understand it? What was a main idea of algebra, or what was the point?

6. Would you describe algebra as easy or hard? Explain your answer.

7. You have already agreed to let me view your grades. Do you remember your grades in your math classes? Would you mind sharing those with me?

8. Do you remember your ACT score? What was your ACT score in math?

9. Here are 10 subject matter areas: art, health, language arts (English), math, music, physical education (PE), reading, science, social studies, and writing. **(*Have these written out for the students to view.)*** What rank would you assign to math if 1 indicates your favorite subject and 10 indicates your least favorite subject?

10. Why did you rank math the way you did?

11. Do you think of yourself as being "good at math" or "not so good"? Why did you answer the way you did?

    Skip 12-14 for non-Hands-On Equations students.

12. Do you think that the Hands-On Equations experience made a difference in your learning math later on in high school or college? If so, how? If not, why not?

13. Do you think other sixth graders should learn Hands-On Equations? Why or why not?

14. Can you give me three facts that you recall about Hands-On Equations?

15. Tell me about two positive experiences in math that you recall. Tell me about two negative experiences in math that you recall.

16. Would you please solve the following equations: ***(Have these written out.)***

$2(x + 4) + x = x + 16$	$2x + (-x) + 3 = 2(-x) + 15$
$2x + x + x + 2 = 2x + 10$	$2x – 3(-x) = 20 + x$
$2(-x + 8) – (-3) = 2x + 3$	$2x – 2(-x + 4) = x + (-2)$

Listed below are 10 subject matter areas. What rank would you assign to math if 1 indicates your favorite subject and 10 indicates your least favorite subject? You do not need to rank all of the other subjects unless that is helpful to you.

______ art

______ health

______ language arts (English)

______ math

______ music

______ physical education (PE)

______ reading

______ science

______ social studies

______ writing

Name____________________ Date______________________

$2(x + 4) + x = x + 16$

$2x + x + x + 2 = 2x + 10$

$2x + (-x) + 3 = 2(-x) + 15$

$2x - 3(-x) = 20 + x$

$2(-x + 8) - (-3) = 2x + 3$

$2x - 2(-x + 4) = x + (-2)$

APPENDIX H

PILOT STUDY TRANSCRIPTION

Pilot Study

Note:
The interviewer's input is in regular text; the interviewee's comments are in **bold.**

Were you in Mrs. Robert's sixth grade class when Hands-On Equations were taught?

Yes.

Can you remember any of your reactions to those lessons now?

I can remember thinking that it was fun. I liked it because it was something new, and I got it really easy, so-- it was interesting to me.

OK. So that was your memory of your reaction then. Did you take algebra in junior high, high school, or college?

Yes.

So what did you take?

I took, uh, Algebra I and II, well, I took Algebra I in eighth grade and then I took Algebra II and Advanced Math I and II so that was pre-calc.

That was still high school?

Yeah.

That was pre-calc. OK. And what have you taken since then?

I've taken Calculus I.

Just I?

Uh-huh.

Describe your experiences when you got to your first algebra class.

At first it was a little bit overwhelming. I can remember the first couple of weeks just being kind of lost after we got done with the review of everything we'd done. But as soon as I got over the initial—like, kind of angst about it, I think it went really smoothly.

Hmm. So where do you think that came from, that initial. . .

I don't know. I was just. . .probably I hyped it up in my head thinking this is tough math now.

So that was eighth grade algebra?

Um-hm.

Would you describe algebra as easy or hard?

Easy.

Explain your answer.

It just makes sense to me. It's really easy for me to do what does this variable mean. If it's geometry, it's a way different story. I've just always been able to get algebra quickly.

You have already agreed to let me view your grades. Do you remember your grades in your algebra classes? Would you mind sharing those with me?

I got A's in Algebra II, I think, maybe one B+. Geometry I think was mostly Bs. And then Advanced Math I, I got mostly B+s and then in Advanced Math II, I got A-s all the way through.

Do you remember your ACT score? What was your ACT score in math?

I think it was a 30 or 28, 32 maybe. 32 was one of. . . It was even. An even number.

What do you think about math? Do you like it or dislike it?

I *liked* math until I got to Calculus and then I felt a little bit lost. But I think everybody feels a little bit lost.

And that was in college?

No, in college math I was fine because it was kind of a review of the advanced math stuff that we did.

OK, so you're talking about the calculus you had in high school?

Yes, the first calculus class that I took just kind of deterred me and geometry was not fun for me because I didn't like the proofs. I'm not as visual with the lines and everything…it didn't work for me.

So. . . so, overall, do you like it, or it depends on what it is, is what I'm hearing you say. It depends on what math it is whether you like it or not.

I love to factor. Factor things out, foil and kind of work your way down to the. . .but besides that, it's. . .I just think that's fun.

Why did you respond the way you did? Is there anything else you can think to say that would explain why you responded the way you did?

I don't know. Umm, I guess it would have been easier if maybe I had taken it a little bit slower, if I'd had just a little bit more instruction, because I can remember in Mr. Herpich's class, he'd assign us problems and it was really kind of before we had talked about them. We'd talk about them after we had done the assignment. And I didn't think that was very helpful. There were times in my Advanced Math II class which is kind of like Calculus I and II, of, he would assign us problems and we'd ask him the next day how to do it, and he'd be like, "Oh, this is a hard one. I don't know how to do this, actually."

So how did you respond to that?

It did deter me from doing, from really stressing myself, from going as far with myself as I could have. It made me feel like, well, if I don't get it in the first, you know, ten minutes, then I'll just stop because he probably won't know how to do it either. I kind of gave up easier. It made it easier for me to stop and say, "Well, it's okay that I don't get this.

Do you think of yourself as being "good at math" or "not so good"?

I think I'm fairly good at math.

Do you think that the Hands-On Equations experience made a difference in your learning math later on in high school or college? If so, how? If not, why not?

Hm. I think it helped in the beginning when I first started to do more higher math.

And how was that?

More like Algebra I and II. I think it made that transition a lot easier.

OK. Tell me about that.

It was just a lot easier to think of like, the little pieces.

So, you were thinking of those. . .?

Yeah, with the simple beginning problems, it was easy to imagine that and to think, well, it gave you something to visualize. And it wasn't just marks on a piece of paper. You could actually think about it in your head, and picture it and it made it a little more real life as opposed to. . .

So you were picturing the manipulatives that we used? Is that what you are saying?

More or less.

Or were you picturing conceptually what was happening?

Conceptually. Yeah. Because with calculus, or things like that, if you think about. . .or even geometry, if you think about sine and cosine, you can think of the opposite over hypotenuse equals adjacent. Like that's a visual thing that you can actually relate and gives you kind of a reference point. And with the Hands-On Equations, it gave you a reference point for this abstract math that. . .that it wasn't just numbers in your head, it was. . .you could think about it as an actual problem with this little mystery box that you had to figure out what it was.

What do you mean by mystery box?

That's the variable, the little pieces. Trying to figure out what the value of that one. . .the mystery was.

Do you think other sixth graders should learn Hands-On Equations? Why or why not?

Yes.

Then, why?

I think it makes learning that kind of stuff. . .for me, it made it funner and it made it easier and it was math that I didn't mind doing.

You minded math before that?

It depended. If we were doing something like word problems, or something that would be just busy work like add up long problems or do long division, something like that, it was more of a chore. But this was really kind of fun and investigative and kind of sparked. . .it made you want to figure out what it was. It was intriguing.

OK, so what was it about it that made you want to figure it out? Again, now you're talking about the Hands-On Equations materials?

Um-hm. It was just something new. It was new, and I liked the way that I got it. It seemed really quick.

Do you have any other comments about Hands-On Equations?

Not that I can think of right now.

OK. It's been a long time ago. Do you have any other comments in general about learning math?

Hm. I've run into a lot of experiences where professors or teachers are kind of. . .it's hard to explain math is what I've come to the conclusion. It's hard to. . .

Do you mean in general or just for some people?

I don't know. To a lot of people, if you get it, then you're fine and you can breeze through anything but if you don't just get it right off the cuff, if you just don't understand the process, then a teacher's got to be really good at explaining what's happening. I remember just beginning calculus, it was really hard to know, hey, why do I do this? Why don't I know this little trick that seems so easy for this person over here? And I felt like I was just kind of missing something.

Were you looking for the trick or were you looking for understanding?

Understanding. Just something that brought this math to a level where I could say this is why you do this, this is what this relates to, this is what this math is about. And it wasn't just kind of stabbing in the dark like performing different functions just because you kind of thought you were supposed to. Like there was no form way to solve everything with derivatives. It's. . .there's an easy little formula and you can do that for a lot of simple equations but then when you get to the higher level ones, you've got to figure things out, factor things out, figure out these little sets like sine and cosine, this one, or something like that. You can. . .there are just so many little catches and it takes somebody. I don't know if you just have to be skilled or what, but you have to have a way with words for a teacher to really explain it in a way that everybody sees what's actually going on.

That's what's called good teaching.

Yeah. And that seems hard to find like even at my level.

For math, you mean?

Yeah, there's still. There's some people who just don't get it and they just need more explanation. A lot of times, teachers skip steps because obviously, they get it.

Right. I know what you're talking about—that skipping steps. It's like if you can't—you've got to construct the steps in your mind in order to connect what's in between, and if you can, you're okay, and if you can't, you're like, "how did you get there?"

Yeah, for one person, the teacher will be like, "Oh, it's simple. You just do this." Whereas in that one little suggestion that the teacher makes, there are three or four steps piled up where somebody's like, whoa, wait a minute. I don't see how you get from point A to point B so easily.

You took calculus in college and that was it?

Yeah.

So are you done with your math?

I was going to take Calculus II but then I decided against it because I don't really need it.

You don't need it for your major?

Um-hm.

Which is?

Anthropology.

Oh, anthropology. I thought it was biology.

Yeah, I stopped because I am really bad at chemistry.

I wonder if you really are.

That's one of those times when I just don't get the little tiny steps.

Would you please solve the following equations:

$2(x + 4) + x = x + 16$	$2x + (-x) + 3 = 2(-x) + 15$
$2x + x + x + 2 = 2x + 10$	$2x – 3(-x) = 20 + x$
$2(-x + 8) – (-3) = 2x + 3$	$2x – 2(-x + 4) = x + (-2)$

He quickly worked the problems and got 6 out of 6 correct. This interview took 14 minutes on the tape plus the 1-2 minutes he took to complete the six problems and the preliminary visiting.

APPENDIX I

DESCRIPTIVE DATA ON STUDENT PARTICIPANTS

Appendix I - Table 1

Descriptive Data for Hands-On Equations Students

Student	Gender	GPA	ACT Score in Mathematics	Special Services
1	F	4.00	28	
2	M	3.84	32	Gifted
3	M	3.43	22	
4	M	3.06	NA	
5	M	2.97	17	
6	F	2.94	16	
7	F	2.89	17	LD
8	M	2.70	17	
9	F	2.64	16	
10	M	2.13	NA	

Note. NA = no score for the ACT; Gifted = Identified as gifted; LD = Identified as learning disabled.

Appendix I - Table 2

Descriptive Data for Non-Hands-On Equations Students

Student	Gender	GPA	ACT Score in Mathematics	Special Services*
11	F	4.00	27	
12	F	3.88	31	
13	M	3.49	29	
14	F	3.44	16	
15	M	3.28	25	
16	M	2.93	21	LD*
17	F	2.75	15	
18	F	2.58	NA	
19	M	2.14	NA	

Note. NA = no score for the ACT; *Special services information was not available for the non-Hands-On Equations students. The LD or learning disabled designation was self-reported by Student 16.