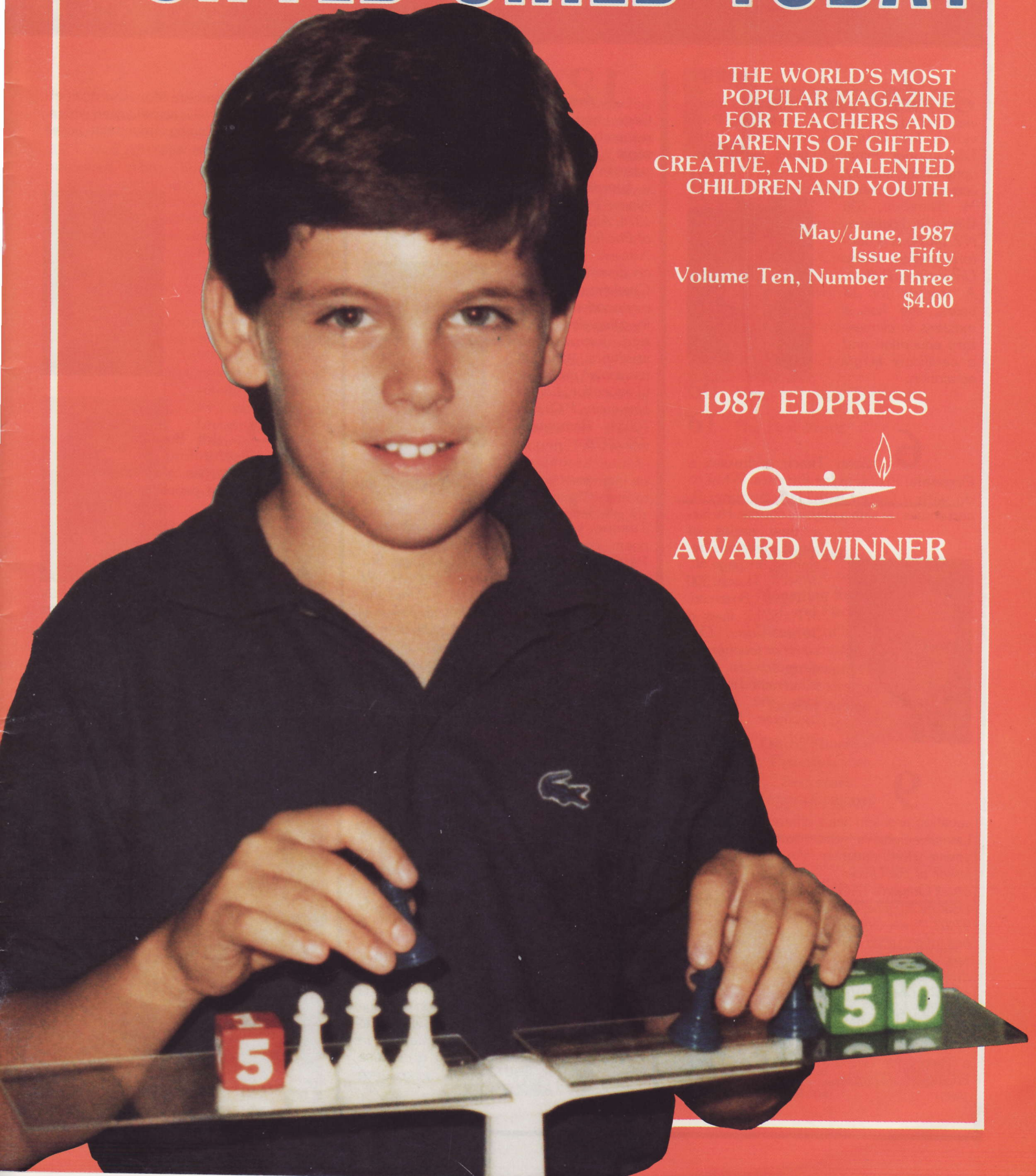


THE

GIFTED CHILD TODAY



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PARENTS OF GIFTED,
CREATIVE, AND TALENTED
CHILDREN AND YOUTH.

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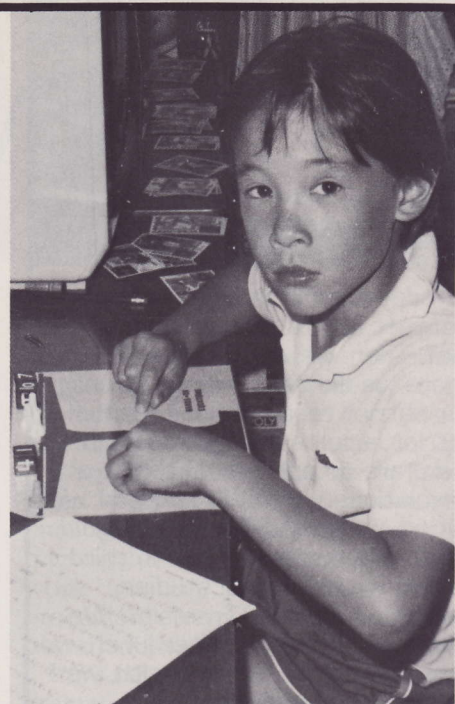
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AWARD WINNER

Algebra for Gifted Third Graders

by Henry Borenson



In several schools and homes in the Bucks County, Pennsylvania area, g/c/t third graders and average fourth graders have demonstrated a strong interest in learning ninth-grade algebraic concepts. Through a novel approach, *The Hands-On Equations™ Learning System*, these children are able to solve such algebraic linear equations as:

$$2(x + 4) + x = 2x + 10$$

and

$$2x - 3(-x) = 20 + x.$$

This article presents a rationale for that program as well as an overview of its scope, methods, and objectives.

Though beneficial for all school children, the system is ideal for use with elementary g/c/t or enrichment classes, third grade and up. It should be noted that the program is not necessarily an acceleration program, although it may be used with precocious children as a first introduction to algebraic concepts. Rather, the program is designed to

provide young children with a successful introduction to important algebraic concepts and methods through a game-like, hands-on, concrete approach to the subject.

Designed to stimulate a young child's mathematical interests and enhance a sense of self-esteem, the system works by creating a physical model of the abstract, formal world of algebraic linear equations. And since young children function best at the concrete operational level, they can easily experience success with equations and concepts which may stump many ninth graders who are taught by the traditional abstract methods.

By providing young children with successful introduction to algebraic notation, methods, and concepts, the stage is set for later success in formal algebraic studies. For the g/c/t child, the program may represent the child's first introduction to a mathematical system involving not only addition, subtraction, multiplication, and division but also problem solving and logical and strategic thinking. Many g/c/t children are challenged mathematically by the program, some perhaps for the first time in their school career. Others find

that what may have been a lagging mathematical interest has been stimulated anew.

The program may be used by elementary g/c/t teachers or by a parent working individually with a g/c/t child. One of the interesting discoveries made in implementing the program was that some g/c/t children in the second, third, and fourth grades were able to learn the program essentially on their own by reading the manual and following along with their game pieces. Elementary school teachers who have used the program have done so without the need for any inservice. Said one third-grade g/c/t teacher, who used the program with her class, "It (the program) eased my fears of algebra!"

The experience of these children and teacher confirms the simplicity of the 25 steps or lessons, encompassing three levels, which take a young child having no formal algebraic prerequisites to the point where he/she is able to solve easily and enjoyably such algebraic linear equations as:

(Level I)	$2(2x + 3) + x = 3x + 10$
(Level II)	$2x - 3(-x) = 20 + x$
(Level III)	$2x - 2(-x+4) = x + (-2)$

One class of fourth graders, containing students with a wide variety of abilities in math, was able to complete the program by doing one lesson every 1 or 2 weeks. Most lessons can be completed within a half-hour time period, and this includes giving students an opportunity to solve the worksheet problems at their desks using the student kits. Some g/c/t students as young as second and third graders have been able to learn each level directly from the manual within the remarkably short time of one hour per level!

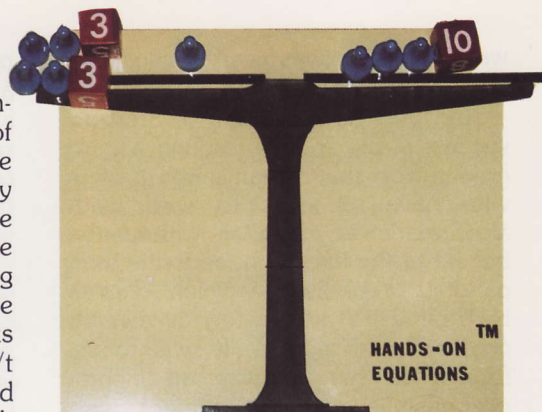
It should be noted that these children are not only able to solve correctly these ninth-grade algebraic linear equations, they are also able to verbalize what they are doing and the logic behind it. In other words, through these concrete methods, the children are developing a sound mathematical sense about algebraic linear equations and how they work.

Gordon Chin, a 7-year-old now going into the third grade, was able to learn all three levels on his own. He had to read only a few of the lessons before he was able to use those concepts to figure out and discover other moves which enabled him to solve the more advanced equations. But Gordon is an unusual second grader. And although he, and other second graders have shown success with the program, it is nonetheless recommended that the program be used with g/c/t children who are at least in the third grade.

This series of illustrations conveys how children physically represent and then solve algebraic linear equations using the *Hands-On Equations*TM system. The equation chosen is the equation selected above from Level I:

$$2(2x + 3) + x = 3x + 10$$

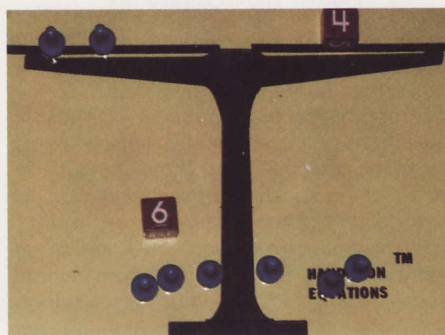
In the first step of the process, the child physically "sets up" the equation using blue pawns and red numbered-cubes. We note that the blue pawns represent x 's and the red numbered-cubes represent the numerals in the equation. Gordon has set up two blue pawns and a red 3-cube to represent $2x + 3$, and then he has doubled that to represent the 2 outside the parenthesis of $2(2x + 3)$; then he has added another pawn to complete the left side of the setup. On the right, he has set up three blue pawns and a red 10-cube to represent $3x + 10$.



In the next step, Gordon uses a *legal move*: he removes three pawns from the left side of the setup and three from the right side of the setup, leaving a simplified setup on the balance.



Finally, Gordon removes a 6 red-cube value from each side of the setup, leaving only two blue pawns on the left side, and a red 4-cube on the right.



From here, Gordon easily sees that each pawn has a value of 2. Hence the solution to the initial equation is $x = 2$. Next, Gordon checks his answer by setting up the initial equation one more time. Since each pawn is worth 2, the left side is worth 16 and so is the right side. Gordon then writes, "Check $16 = 16$."

This example, taken from the early part of the program, conveys the flavor of the approach: The children physically represent given equations using pawns and numbered cubes; they then solve them using a series of physical *legal moves*; and finally, they check their solutions in the initial setups.

Similar methods are used with the more advanced equations of Levels II and III. In these levels, white pawns are used to represent $-x$ and green numbered cubes are used to represent negative integers. The setup for the equation

$$5 + 3(-x) + x = 2x + (-15)$$

is shown on the cover of this issue using the teacher's demonstration scale and demonstration game pieces.

It will be noted that with this program a child physically carries out what a student in a traditional ninth-grade algebra course would carry out abstractly. We saw, for example, that Gordon removed three blue pawns from each side of his setup instead of subtracting three x 's from each side of the abstract equation. Hence, the child is building a sound sense of important algebraic principles, such as the subtraction property of equality (although this term is never used) through concrete means. Furthermore, the child finds learning about algebraic linear equations through these methods to be quite natural. Seymour Papert, (1980, p.7) in his classic book *Mindstorms*, refers to the concept of "Piagetian learning." This is the natural kind of learning through which we all, for example, learn to speak as young children. In a similar manner, a hands-on system represents a physical *microworld* enabling the young child to acquire easily and naturally significant algebraic concepts.

Another important benefit of the program is worth noting. Steven Bilheimer, age 9, was also able to learn the entire program on his own by reading the manual, while in the third

grade. When presented with the Optional Lesson #26, which interrelates the concrete and pictorial methods of the program with the traditional written methods for solving algebraic linear equations, Steven had merely to read one example of the correspondence. He was then able to solve all the other equations of the lessons without resorting to either the pawns or the pictorial notation introduced in the program.

Steven's experience vividly illustrates a principle which Maria Montessori referred to as "the spontaneous rise from the concrete to the abstract" (Standing, 1966, p. 51). In other words, Steven's hands-on, concrete, and pictorial experiences with algebraic concepts were such that he needed very little lift to enable him to rise to the abstract level of solving algebraic linear equations. His concrete experiences had provided him with a strong and sound understanding of fundamental algebraic concepts and methods. To move to the traditional

written solution was a very insignificant step for Steven.

And yet, how different is Steven's successful experience, though a third grader, from that of many ninth graders who have to learn to work with algebraic linear equations without the benefit of the hands-on, concrete, and pictorial experiences which Steven had? Steven's own sister, as a ninth grader, failed algebra. It was totally foreign to her. It was an incomprehensible, abstract subject with rules, making very little sense, which had to be memorized — and memorization was not her strength. She had never had any preparation for algebra in the earlier grades.

Often, at the ninth-grade level, teachers do not consider concrete methods appropriate. They are not aware of the tremendous power concrete methods offer. So, the preferred model for teaching a concept (concrete → pictorial → abstract) is bypassed in most ninth-grade algebra classrooms. The students are

presented at once with concepts at the abstract, formal level. For those students who cannot easily memorize rules and carry them out as per the teacher's directions, algebra as traditionally taught can be a difficult subject.

Elementary school children using a successful, hands-on, experience in algebraic linear equations can develop a positive mind-set and expectation for success in later formal, algebraic studies. For the g/c/t child, such a hands-on approach can open up the world of algebraic linear equations for all its fascination, challenge, and mystery.

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From Our Readers:

Let me tell you, GCT

Reader Ginsberg Writes

The Gifted Child Today
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Mobile, Alabama 3660-0448

I don't care that you chose to change the publication's name. I do care that you've changed your standards!

I have trouble recalling (and I was a charter subscriber!) an article so full of puffery and lacking in substance as Sylvia B. Rimm's "Marching to the Beat of a Different Drummer" (January/February, 1987). To compound the insult, her article is merely a thinly-veiled attempt to advertise her own materials — an attempt that will surely backfire if one assumes that her materials are as vacuous as her article.

Speaking of vacuous articles, what was the purpose of including Richard Bothmer's "Top Plays of the Summer?" Some of us work at delivering services to children rather than at traveling from Guam to workshops all over the mainland (probably at taxpayer expense!).

Of equal concern is the profusion of typographical and usage errors that appear

in your magazine with increasing frequency. Some examples from your [same] issue: *conscious-raisers* for *consciousness-raisers*, *visably* for *visibly*, *devestating* for *devastating*. . . .

I trust that you get my point, even though I haven't yet finished reading the entire issue. A magazine dedicated to excellence should promote it through exemplars in both substance and form.

Robert A. Ginsberg, Supervisor
Programs for the Gifted and Talented
East Brunswick, NJ

Author Rimm Responds

Marching to the Beat of a Different Drummer — Surely Mr. Ginsberg must be criticizing some other article. Those children are real and represent a sample of the many highly creative children we help at the Clinic. Two of those three cases upon which the article is based continue to perform up to their ability four years after their discharge from very brief individual and parenting therapy. The high school student improved also, but her situation was more extreme. However, she did

graduate from high school instead of "dropping out" which was one of our initial concerns. We help dozens of such children on a regular basis and want to share our techniques. Why is that puffery?

As to your accusation of advertisement for GIFT and GIFFI, these are carefully researched inventories which we use in our Clinic and which are used in many schools. They are described in two sentences, less than 1% of the article — that's a 99% thick veil. Vacuous is a good word for describing empty criticism. More important, however, I feel some distress that a gifted program supervisor is not more sensitive and aware that the cases I've described represent real problems of creatively gifted children.

Sylvia Rimm, Director/Psychologist
Family Achievement Clinic
Oconomowoc, WI

Author Bothmer Responds

Top Plays of the Summer — Surely it is not the letter writer's position that workshop participation is a frivolous activity which steals time from "delivering