The Effect of Hands-On Equations® on the Learning of Algebra by 4th and 5th Graders of the Broward County Public Schools

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Hands-On Equations Interim Report: March 17, 2008
A Study of the Strength of Acquisition of Algebraic Concepts by 4th and 5th Graders
Via Hands-On Equations and a Measure of the Retention of the Pictorial Notation

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ABSTRACT

The Broward County Public Schools agreed to participate in a research study to determine the effectiveness of the Hands-On Equations® program in providing its students with a successful experience with algebra. The study sought to determine whether the 4th and 5th grade students of the district could learn to solve equations such as $3x = x + 12$ and $4x + 3 = 3x + 6$, equations normally presented in the 8th or 9th grade. If the students were successful with these concepts, they would have overcome at an early age one of the obstacles to the learning of algebra.

The teachers who participated in this study received a full day of training in the use of the program. The workshop they attended, the Making Algebra Child's Play® workshop, was conducted by a certified Borenson and Associates, Inc. instructor in the fall of 2007. Immediately after instruction, the teachers administered a pre-test to their students, and then proceeded to teach the first seven lessons (Level I) of Hands-On Equations. They also administered two post-tests and a three-week retention test.

This report presents the meta-analysis conducted on six 4th grade regular classes, three regular 5th grade classes and five gifted and talented 5th grade classes, a total of 14 classes involving 326 students. The Appendix includes the test results for other classes participating in the study. For various technical reasons explained in the report these additional classes could not be included in the meta-analyses.

Since the teachers and students participating in this study were representative of those in the district as a whole, the results shown herein are indicative of the results that would be expected if the Broward County Public Schools were to implement the program district-wide in the 4th and 5th grades.

The authors wish to thank Miriam Sandbrand, Mathematics Curriculum Specialist, K-5, for her efforts in coordinating this study and to the teachers who participated in this study.

GENERAL INTRODUCTION

HANDS-ON EQUATIONS®

Hands-On Equations (HOE), is a program developed by Dr. Henry Borenson (one of the authors of the present study), to provide an intuitive, hands-on approach, to presenting algebraic concepts to grade school and middle school students. The program uses numbered-cubes to represent the constants, and blue pawns to represent the variable ***x***. It also uses a scale representation on which the students "set up" the equation. A typical setup is shown in Figure 1.

Hands-On Equations Representation of $4x + 2 = 3x + 9$

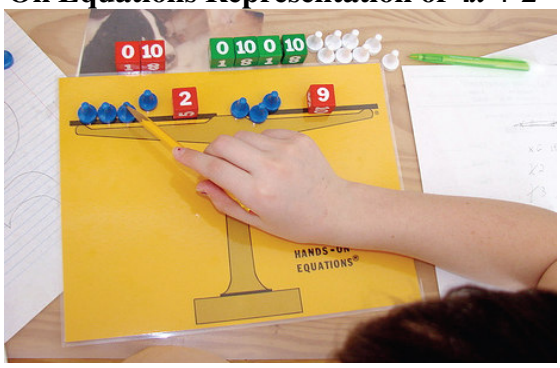


Figure 1

The students then proceed to use "legal moves," which are the mathematical counterpart of the abstract algebraic methods which are used to solve these linear equations. The system thus makes abstract linear equations visual and understandable, and further provides students with the means of solution through a kinesthetic approach which makes sense to them. An example of this approach is shown in Appendix 1.

The program is unique in that the abstract knowledge base needed by students to solve these equations is transformed into an easily understood and manageable set of verbal, visual and kinesthetic responses using manipulatives. The program teaches algebraic principles which students in grades 3 to 8 can apply in any sequence desired to solve the given equation. Hence, the students using Hands-On Equations need not memorize a series of steps to solve an equation, as is the case in more traditional instruction. Rather they feel empowered to use their thinking and understanding of basic principles to solve the problem at hand. (See Appendix 2 for the objectives of Level I of the program)

META-ANALYSIS APPROACH

The research studies mentioned in this report, as well as the series of studies of which this is a part, use a multi-site replications design and a meta-analysis procedure to study the effect of the HOE program on many groups of students with different characteristics (regular education students, special education students, elementary, middle, and high school students, inner city, rural, suburban, gifted and handicapped). Some of the above groups will be studied separately. In other cases, the classroom with a diverse student population will be studied as a unit. Similar groups will then be combined into a larger study, thus the meta-analysis component of this design. Presently we have data on more than 90 classrooms in 13 states involving over 1,876 students.

DEFINITION OF TERMS

The definitions below will clarify the nature of each of the tests, as well as the various terms used in this report.

Blue Pawn: The student game pieces which are used to represent the variable x. $4x$ for example, would be represented by 4 blue pawns. (See Fig. 1 which is on the previous page).

Red Numbered Cubes: The student game pieces used to represent the positive constants. The expression $4x + 2$ would be represented by 4 blue pawns and a cube with the number 2 displayed. (See Fig. 1)

Flat Laminated Balance: A representation of a balance scale printed on paper, laminated for protection. The students set up their equation on the flat balance scale. (See Fig. 1)

Game Pieces: When students solve the equations using the manipulatives that come with the program, namely, the blue pawns, the number cubes and the flat laminated balance, we will say that "the students solved the equations using the game pieces."

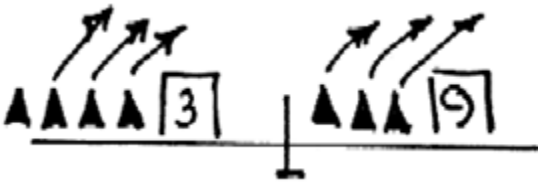
Teacher's Balance Scale and Game Pieces: A stationary balance scale and game pieces used by the instructor in the front of the room to illustrate the equations.

Setup: The set of all the pieces that are placed either on the student balance scale or on the Teacher's Demonstration Scale to represent the algebraic equation.

Legal Move: These are the moves a student may perform and still keep the equation in balance. In Level I of HOE, which is the subject of the present study, the legal move is the Subtraction Property of Equality. In particular, the students may subtract the same number of pawns from both sides of the scale, or they may subtract the same cube value from both sides of the scale. Referring back to Figure 1 on the previous page, the students may subtract three blue pawns from each side of the setup to obtain the result from what is left. If they wish, they may also take away a 2 value from the cubes on both sides.

Kinesthetic: This term is used to indicate that the simplification of the equation is carried out in a physical manner. For example, in removing a pawn from each side of the balanced system, the student is developing a bodily sense of this legal move. Hence, the concept is being learned at a level similar to learning a dancing or gymnastic move. The student gains a bodily sense of the mathematical concept, and hence the term kinesthetic indicates the use of the additional learning modality which involves body memory.

Pictorial Notation: Once the students have learned to solve the equations with the game pieces, they learn to solve the equations using only paper and pencil by drawing pictures of the game pieces, the balance scale and the legal moves. The pictorial representation and solution to $4x + 3 = 3x + 9$ is shown below and described in more detail in Appendix 3.



DESCRIPTION OF TESTS USED IN THE STUDY

Each of the tests used in this study had six questions of increasing difficulty. Each of the questions on each of the tests was randomly selected from a pool of similar questions, each designed to test the student's ability to solve a particular type of equation. The test items were non multiple-choice. The students were to find the value for x and the value for the check. The scoring of each question, however, was only based on the value for x. The students were provided with 15 minutes to respond to each of the tests. Samples of all of these tests are provided in Appendix 4. Below is a description of the various tests used in this study, as well as the abbreviations that will sometimes be used to refer to them.

P: Pre-test. A test administered to students prior to their introduction to HOE.

P6: Lesson #6 Post-Test. A post-test in which the students were free to use the game pieces, flat laminated balance, and the methods learned in Lessons #1 - #6 of HOE to solve the equations.

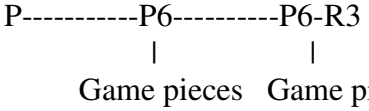
P6-R3: Lesson #6 Three-Week Retention Test. A retention test to determine to what extent the students maintained the concepts learned in Lessons #1- #6 of HOE following a three-week period of no HOE instruction. For this post-test, the students were allowed to use their game pieces. P6-R3 was a retention test used in a prior study.

P7: Lesson #7 Post-Test. A post-test in which the students were not allowed to use their game pieces, but could use the pictorial notation learned in Lesson #7 along with the concepts learned in the first six lessons of HOE. This post-test was used to determine the extent to which the students were able to move away from the use of the game pieces for solving equations to the pictorial system using only paper and pencil.

P7-R3: Lesson #7 Three-Week Retention Test. A retention test administered three weeks after the Lesson #7 Post-Test The students were not allowed to use their game pieces on the retention test, but could use the pictorial notation learned in Lesson #7 along with the concepts learned in the first six lessons of HOE. This retention-test was used to determine the extent to which the students were able to solve equations using the pictorial notation after three weeks of no HOE instruction. P7-R3 was a retention test used in the current study.

PRIOR STUDIES

Study #33c A previous study, #33c, conducted with 194 inner city 5th grade students, measured the level of achievement on the post-test following Lesson #6 using the game pieces, and then again after a three-week period of no HOE instruction, at which time the students were provided with a retention test also using the game pieces. Schematically, we have



That study found that the gain from the pre-test to the post-test was very large and significant (in percentage terms the gain increased from 42.8% to 84.7%) with a t value of 22.26; The study also found that the gain from the pre-test to the retention test was very large and significant (in percentage terms, the gain increased from 42.8% on the pre-test to 79.3% on the retention test) with a t value of 17.94. These results are summarized in the table below:

	Pre-test	Post-test after Lesson #6	3-Week Retention Test after Lesson #6
Grade 5, n=194 **Study #33c**	**42.8%** **(m=2.57)**	**84.7%** **(m= 5.08)**	**79.3%** **(m= 4.76)**

t (P, P6)=22.26 **t (P, P6-3R)=17.94**

A t-test designed to measure whether the change in score from P6 (84.7%) to the retention test score (79.3%) yielded a t-value of 3.88, which was significant.

Post-test after Lesson #6	3-Week Retention Test after Lesson #6
84.7% **(m= 5.08)**	**79.3%** **(m= 4.76)**

t(P6 P6-R3)= 3.88, sig @.01

This showed that following three weeks of no instruction the students had indeed lost some of the ability to solve the algebraic equations, and that this loss was statistically significant. Nonetheless the gain from the pre-test score (42.8%) to the retention test score (79%) produced a highly significant gain with a t-value of 17.94.

Study #59a, conducted with 123 4th graders, half of whom were inner city students and half of whom were suburban students in another large school district in the southeastern United States, measured the level of achievement on the post-test following Lesson #6 using the game pieces, and then again after the students had been instructed in the pictorial notation in Lesson #7, and compared those results to see if there was a significant difference. Schematically, we have

P----------P6----------P7
| |
Game pieces Pictorial notation

That study found that the gain from the pre-test to the Lesson #6 post-test was very large and significant (in percentage terms the gain increased from 30% to 84%) with a t value of 22.62; The study also found that the gain from the pre-test to the Lesson #7 post-test to be very large and significant (in percentage terms, the gain increased from 30% on the pre-test to 88% on the Lesson #7 post-test) with a t value of 29.70. These results are summarized in the table below

Study #59a	Pre-test	Post-test after Lesson #6	Post-test after Lesson #7
Grade 4, n=123	**30%** **(m=1.81)**	**84%** **(m=5.04)**	**88%** **(m=5.32)**

t(P, P6) = 22.62 t(P, P7) = 29.70

In order to determine if there was any significant difference in achievement in going from the use of the game pieces for solving equations to the pictorial notation method, a t-test was done between the Lesson #6 post-test scores and the Lesson #7 post-test scores. The t-test was significant, with a t-value of 2.86.

Study #59a

Post-test after Lesson #6	**Post-test after Lesson #7**
84% **(m=5.04)**	**88%** **(m=5.32)**

t(P6, P7) = 2.86, sig@.01

In this study, 59a, we discovered that the students had made a small but significant gain in going from the use of the game pieces to the pictorial notation method of solving equations. Other studies, such as 102b with 192 6th graders and study 105a with 105 8th graders, did not show a significant difference between these two post-tests. In no instance did we see a significant decrease in going from the game pieces to the pictorial notation.

RATIONALE OF BROWARD COUNTY STUDIES

Although Study 33c provided a result regarding the retention of the algebraic concepts after a three-week period without HOE instruction, that study involved the use of the game pieces in the retention study. The students of study #33c were not presented with the pictorial notation, as the intent of that study was to determine the level of acquisition of algebraic concepts using the game pieces, and their level of retention three weeks later, again using the game pieces.

Since generally speaking students will be expected to solve algebraic linear equations without the use of game pieces, we wished to conduct a retention study on the pictorial notation. In order to do such a study, we needed to administer P7, the post-test following Lesson #7. It is in Lesson #7 that the students learn the pictorial notation. We also needed to administer P7-R3, which is a retention test three weeks later. Both the P7 test and the P7-R3 test were tests in which the students were not allowed to use the game pieces. If we had only administered these two tests, along with the pre-test, to determine the level of acquisition and retention of the pictorial notation, the study would look like this schematically:

```
P----------P7----------P7-R3
           |           |
       Pictorial   Pictorial
```

However, we could not simply administer these three tests and omit P6, the post-test after Lesson #6 in which the students used the game pieces. Had we omitted P6, and had the students done poorly on P7 (The Lesson #7 post-test in which the students use the pictorial notation), we would not have known the cause of the low performance on P7: We would not have known, for example, whether this low performance was due to the fact that these students had found the pictorial notation difficult or whether it was due to the fact that the students had never learned to solve the equations, even with the use of the game pieces. In other words, a poor Lesson #7 post-test score might mean that the students had not established the foundation with the game pieces upon which to build the pictorial notation. Hence, in order to conduct testing on the pictorial notation – including its retention level-- we had no choice but to also administer P6 using the game pieces. Schematically, then the current study looks like this:

```
P----------P6----------P7----------P7-R3
           |           |           |
       Game Pieces  Pictorial   Pictorial
```

Furthermore, by administering P6 we would have data for a study very closely resembling studies 59a, 102b and 105a. Schematically, those studies looked like this:

P----------P6----------P7
| |
Game pieces Pictorial notation

Notice that this sequence of tests comprises the first three tests of the present study. The only difference between those studies and the current one is that the current one has an additional test, namely a retention test at the end of the sequence. Nonetheless, the current study would provide another opportunity to test a) the level of acquisition of the concepts of the program as measured by P6, using the game pieces, as well as the level of retention in moving from P6 to P7, with the latter being taken without the use of the game pieces.

PURPOSE OF THE STUDY

The purpose of the current 4th and 5th grade studies was four-fold, as noted below. The last two elements would provide for the first time research data on the retention level of the pictorial notation.

a) First we wished to determine the effectiveness of the HOE program upon these 4th and 5th grade students as measured by the gain in score from the pre-test to P6, which used the game pieces. A t-test would be administered to determine if the gain was statistically significant.
b) Secondly, we wanted to see if the scores obtained on P6, using the game pieces, were maintained on P7, without the use of the game pieces. A t-test would be conducted to determine if there was any significant difference between the P6 and P7 scores.
c) Thirdly, we wanted to see if the scores obtained on P7, without the use of the game pieces, were maintained on the retention test, without the use of the game pieces, following a three week period of no HOE instruction. A t-test would be conducted to determine if there was any significant difference between the P7 and P7-R3 scores.
d) We wanted to measure the gain from the pre-test to the retention test to determine if it was statistically significant.

BROWARD COUNTY FOURTH GRADE STUDY #131MA
Regular Students

(Please read the General Introduction on pages 1 - 7 prior to reading the section below.)

Six 4th grade classrooms, consisting of 111 students, were analyzed together in meta-analysis #131MA.

TEACHERS OF THE STUDY

The teachers participating in this study were selected by the mathematics supervisor from among those who responded to an invitation sent by her to each school, inviting teachers to participate in this research project. Preference was given to teachers who had never taught HOE, who did not have a class set of materials, and who would be willing to train other teachers in their school. Preference was given to two teachers who had previously assisted with district math training. The study submitted by one of these teachers could not be used in this meta-analysis since we had no way to explain retention test scores that were significantly higher than the last post test the students had taken; the other study did not exhibit any unusual features and so was included in this meta-analysis.

According to the mathematics supervisor, "All the teachers and students are representative of the district. The teachers had varied levels of teaching skills, from very experienced to less experienced. There were students from struggling schools, middle-of-the-road schools, and high level schools. It was a well rounded group of teachers and students."

None of the teachers in this study were new or beginning teachers. Three had 5 to 10 years of teaching experience; three had more than 10 years of teaching experience. For each of these teachers this was their first experience teaching HOE. All six teachers stated that they taught the program as instructed and that they made no changes in the teaching procedures.

STUDENTS OF THE STUDY

The students in this study were 111 4th grade students. The 6 teachers described their students as mostly inner city (approximately 65%) and the remaining 35% of the students were described as suburban. The students were a mix of 72 average students, 17 gifted students, 7 LD students and 15 ELL. None of the students had been exposed to HOE prior to this study.

CLASSES OF THE STUDY

Six of the summary forms submitted by the 4th grade teachers fit the criteria to be included in this meta-analysis. In each of these six classes the appropriate pre-test, post-tests and retention tests were administered. Four of the 4th grade classes could not be included in this meta-analysis since they did not meet at least one of the testing conditions of the study. In one instance, we were not able to explain a substantially higher score on the retention test (P7-R3) than on the Lesson #7 post-test (P7). In another instance, the retention test was administered two months after the intended date. In the other two classes, the teachers used a test format which consisted of 8 questions*, instead of the 6-question format that was intended for this study. (This 8-question format test was inadvertently placed into each of the class sets received by the teachers participating in the study, and so it got confused with the 6-question test format distributed to the teachers at the training session. Borenson and Associates, Inc. bear some responsibility for the

teachers having two different test formats in front of them.) Although these four classes could not be included in the meta-analysis, the scores obtained by these classes are shown in Appendix 5.

CLASSROOM INSTRUCTION USED IN THE STUDY

The teachers were to present each of the first seven lessons of the HOE program as instructed in the training seminar. Each lesson involved the teacher presenting a concept to the class, along with two or three practice examples. The time required for this instructional component varied among the classes, with some teachers requiring 10 minutes and others requiring 50 minutes. The average for the lesson presentation for this group of six teachers was about 27 minutes. Following this learning session, the students were provided with a worksheet to complete. Four of the examples on the worksheet were on the current lesson; six of the examples reviewed concepts learned in prior lessons. The worksheets were specifically designed in this manner so that the students would be reviewing all prior lessons each time they did a worksheet. The time spent on the worksheets varied from 15 minutes to 30 minutes. The average time spent on the worksheets for these six classes was about 23 minutes.

For the first six lessons of the program, the teacher used the Teacher's Demonstration Scale and Teacher Game Pieces to illustrate the equations and concepts. The students used their sets of game pieces and their flat laminated balance for these lessons. For Lesson #7, the teacher illustrated the pictorial solutions on the blackboard or overhead projector, and the students presented their solutions on paper at their desks. The worksheet for Lesson #7 contained four pictorial examples and six review examples using the game pieces.

TESTING PROCEDURE

A pre-test was given to the students before they were exposed to the HOE program. At the conclusion of Lesson #6, the students were provided with a post-test in which they were at liberty to use their game pieces (the pawns, cubes, and laminated scale). The students were then taught Lesson #7, and given a second (different) post-test. This time the students were to take the post-test without using the game pieces. The students, however, were free to use the pictorial notation they had learned in Lesson #7. Finally, three weeks later, following a three-week period with no HOE instruction, the students were given a retention test (different from the two post-tests, but with similar questions). On this retention test the students were also not allowed to use the game pieces, but could use the pictorial notation learned in Lesson #7.

All of the classes were taught by teachers who had participated in a one-day Making Algebra Child's Play® workshop conducted by a certified Borenson and Associates, Inc. instructor. The training of the teachers took place on October 25, 2007. The teachers started teaching HOE to their students almost immediately after the training (pre-test given to individual classrooms between October 29 and November 2, 2007). The first six lessons were taught and the Lesson #6 post-test with the game pieces was administered between November 5 and 16, 2007. The seventh lesson was then presented to the students and the Lesson #7 post-test was administered between November 6 and November 26, 2007. Three weeks later the retention test was administered (between November 26 and December 21, 2007).

This study is a meta-analysis of six separate classroom studies, each involving an intact classroom of 4th grade students. Most of the classes had at least one or two LD students. One class listed 40% of their students as ELL; a second class listed 50% of the students as a combination of LD and ELL; a third class noted 50% of the students as gifted. For the purposes this study, all the students were treated the same way, whether classified as LD, ELL or GT, both for the analysis of each individual class and for the combined group. Three of the classrooms

were in inner-city schools, one was a suburban school, and the other two had a mixture of inner city and suburban students.

STUDY HYPOTHESES: 4th GRADE STUDY #131MA

1. Students would score poorly on the pre-test. This result was expected since normally 4th graders (who do not have HOE) are not exposed to many of the algebraic equations presented on this pre-test. (See all test items in Appendix 4)
2. Students would score very well (in the 85% range) on the post-test after Lesson #6, using the game pieces. There would be a large and statistically significant increase from the pre-test to the post-test means after Lesson #6.
3. Students would score very well (in the 85% range) on the post-test after Lesson #7, in which the students do not use the game pieces, and there would be a large and statistically significant increase in test score means from the pre-test to the post-test after Lesson #7.
4. There would not be a statistically significant loss between the means on the post-test after Lesson #6 using the game pieces, and the post-test after Lesson #7, using the pictorial notation, if there was a loss at all. All prior studies showed either no difference or showed a significant increase in going to the pictorial notation.
5. Students would score lower on the three-week retention test than they did on the post-tests after Lesson #6 and Lesson #7, but we did not know if the loss would be significant.
6. Students would score well enough on the three-week retention test to produce a significant gain over the pre-test score.

RESULTS

Six classrooms were included in this meta-analysis (combined N = 111). Each classroom's data was analyzed independently to provide feedback to each teacher about the performance of their students. T-tests were conducted between the means of the pre-test and the means of each of the two post-tests and the mean of the retention test. For each of the six individual classes in this study, the gain from the pre-test to each of the post-tests and to the retention test was statistically significant. In three of the classes the t-value obtained was more than 10.0, with all of the six classes seeing at least a doubling of the score from the pre-test to each of the other tests. Three of the classes saw a tripling of the score from the pre-test to each of the other tests.

For the combined group of the six classes, the gain from the pre-test to each of the post-tests and to the retention test was large and highly significant. The t values obtained are shown below.

Grade 4, n=111 **Study #131MA**	**Pre-test**	**Post-test after Lesson #6**	**Post-test after Lesson #7**	**3-Week Retention Test after Lesson #7**
	26.8% **(m=1.61)**	**84.2%** **(m=5.05)**	**84.2%** **(m=5.05)**	**81%** **(m= 4.86)**

t(P, P6) = 20.50 t(P, P7) = 20.45 t(P, P7-R3) = 19.49

Additionally, a t-test was conducted between the mean of the post-test following Lesson #6, in which the students used the game pieces, and the post-test following Lesson #7, in which the students did not use the game pieces. This difference of the means was not large enough to be significant for any of the six classes. For the combined group of the six classes, the effect size between the post-test mean (5.05) after Lesson #6, in which the students used the game pieces,

and the post-test mean (5.05) after Lesson #7, in which the students used the pictorial notation, was not significant, with t(P6, P7) = .10

Grade 4, n=111
Study #131MA

Post-test after Lesson #6 with game pieces	Post-test after Lesson #7 without game pieces
84.2% (m=5.05)	84.2% (m=5.05)

t(P6, P7) = .10, not significant

Furthermore, a t-test was conducted between the means of the post-test following Lesson #6, in which students used the game pieces, and the three-week retention test, in which the students used the pictorial notation. This difference of the means was large enough to be significant in one of the six classes. For the combined group of the six classes, the effect size between the Lesson #6 post-test mean of 5.05 in which the students used the game pieces and the three-week retention mean of 4.86, in which the students did not use the game pieces, was not insignificant with t(P6, P6-R3) = 1.83.

Grade 4, n=111
Study #131MA

Post-test after Lesson #6 With game pieces	3-Week Retention Test After Lesson #7 Without game pieces
84.2% (m=5.05)	81% (m=4.86)

t(P6, P7-R3) = 1.83, not significant

Finally, a t-test was conducted between the means of the post-test following Lesson #7, in which students did not use the game pieces, and the three-week retention test, in which the students also did not use the game pieces. This difference of the means was not enough to be significant in any of the six classes. For the combined group of the six classes, the effect size between the Lesson #7 post-test mean of 5.05 in which the students did not use the game pieces and the three-week retention mean of 4.86, in which the students did not use the game pieces, was not significant with t(P7, P6-R3) = 2.29.

Grade 4, n=111
Study #131MA

Post-test after Lesson #7 Without game pieces	3-Week Retention Test after Lesson #7 Without game pieces
84.2% (m=5.05)	81% (m=4.86)

t(P7, P7-R3) = 2.29, not significant

CONCLUSIONS

This study leads to the following conclusions:

1. The fourth grade students in this study did not perform well on the pre-test, obtaining an average score of 26.8%, and confirming hypothesis 1. Study #59, also with 4th graders, yielded a pre-test average of 30%. The most likely explanation for these low pre-test score is that typically 4th graders (not using HOE) are not exposed to most of the algebraic equations presented on this test.
2. These fourth grade students had large and statistically significant increases between the score obtained on their pre-test (26.8%) and that obtained on either the post-test after Lesson #6 (84.2%) or that obtained on the post-test after Lesson #7 (84.2%). These findings confirm hypotheses 2 and 3 and are clearly congruent with study #59a cited earlier, which involved 123 4th grade students. These findings are also congruent with study 102b involving 196 6th graders and study 105a, involving 105 8th graders. In each instance the gain from pre-test to post-test, whether the post-test was administered with the game pieces or without was large and highly significant.
3. The findings of a large and significant increase from the pre-test to the Lesson #7 retention test confirm hypothesis 6. Following three weeks of no HOE instruction, and provided with only paper and pencil (and no game pieces) the students were able to score significantly higher than on the pre-test, increasing their score from 26.8% on the pre-test to 81% on the retention test. This result is consistent with that obtained in study #33c, the retention study using the game pieces, wherein the 5th graders scored 79% on the Lesson #6 retention test, using the game pieces, which was also a significant gain from the pre-test score of 42.8%.
4. The findings of no significant difference between the means of the post-test after Lesson #6, using the game pieces, and the post-test after Lesson #7, in which the students did not use the game pieces, confirm hypothesis 4. In each of the studies 59a, 102b and 105a, the students either achieved the same mean on the post-test following Lesson #7 as compared with the post-test following Lesson #6, or they achieved a slightly higher mean. These four studies, involving a total 529 students, show that students are able to transfer their hands-on learning to the solution of algebraic equations using the pictorial solution and do at least as well on the pictorial as they do with the game pieces.
5. As expected by hypothesis 5, the students scored lower on the Lesson #7 retention test (81%) in which they used the pictorial notation to solve the equations, then they did on the Lesson #7 post-test (85%), in which they also used the pictorial notation. Hypothesis 5 had left unanswered whether this difference would be significant. The present study, hence, gives us the new research result, namely, that the 4th grade students were able to maintain their learning from the first seven lessons of HOE to the point that they could use the pictorial notation three weeks after instruction, and obtain essentially the same results as those obtained immediately after the instruction of Lesson #7, with no statistically significant difference in the results.
6. A t-test between the Lesson #6 post-test using the game pieces (84.2%), and the Lesson #7 retention test P7-R3 without the use of the game pieces (81%), showed no significant difference. This result demonstrated that not only where the students able to transfer their hands-on learning to the pictorial notation, but that their ability to solve the equations using the pictorial notation, even after three weeks of no HOE instruction, was comparable to their ability to solve it with the game pieces immediately after instruction.

SUMMARY: BROWARD COUNTY 4th GRADE STUDY, REGULAR STUDENTS
LEVEL OF ACQUISITION OF ALGEBRAIC CONCEPTS

This study demonstrated that the combined group of 111 4th grade students, more than 60% of whom were inner city students, a) achieved a large and highly significant gain from the pre-test to the post-test following Lesson #6 in which the students used the game pieces, and b) that this significant gain was maintained on the post-test following Lesson #7, where the students did not use the game pieces (rather, they used the pictorial notation learned in Lesson #7).

	Pre-test	**Post-test after Lesson #6 with game pieces**	**Post-test after Lesson #7 without game pieces**
Grade 4, n=111 **Study #131MA**	**26.8%** (m=1.61)	**84.2%** (m=5.05) t(P, P6)= 20.59	**84.2%** (m=5.05) t(P, P7)=20.95

These conclusions replicate those found in study 59a (123 4th graders), 102b (196 6th graders) and 105a (105 8th graders) in which the students were given the same pre-test, P6 and P7 post-tests as the current study, although the retention test was not part of those three studies. For each of those grade groups the gains from pre-test to each of these post-tests was statistically significant, and the gain was maintained or increased slightly as the students moved away from using the game pieces to the pictorial notation.

	Pre-test	**Post-test after Lesson #6 with game pieces**	**Post-test after Lesson #7without game pieces**
Grade 4, n=123 **Study #59a**	**30%** (m=1.81)	**84%** (m=5.04) t(P, P6)= 22.62	**88%** (m=5.32) t(P, P7)=29.70
Grade 6, n=190 **Study #102b**	**48.2%** (m=2.89)	**92%** (m=5.54) t(P, P6)= 25.15	**93%** (m=5.64) t(P, P7)=22.48
Grade 8, n=105 **Study #105a**	**64.8%** (m=3.89)	**87.7%** (m=5.26) t(P, P8)=8.895	**88.8%** (m=5.34) t(P, P8)=9.99

SUMMARY: BROWARD COUNTY 4th GRADE STUDY, REGULAR STUDENTS
LEVEL OF RETENTION OF PICTORIAL NOTATION

The current study demonstrates that the students had a high retention rate following a three-week period of no HOE instruction. There was no significant difference between the Lesson #7 post-test score (84.2%) and the three-week retention test score (81%), both of which were administered without the use of the game pieces.

Grade 4, n=111
Study #131MA

Post-test after Lesson #7 (without game pieces)	**3-Week Retention Test after Lesson #7** (without game pieces)
84.2% (m=5.05)	**81%** (m= 4.86)

t(P7, P7-R3) = 2.29, not significant

Additionally, the current study demonstrates that the students had a high retention rate compared with the Lesson #6 post-test. There was no significant difference between the Lesson #6 post-test score (84.2%) administered with the game pieces and the three-week retention test score (81%), administered without the use of the game pieces.

Grade 4, n=111
Study #131MA

Post-test after Lesson #6 (with game pieces)	**3-Week Retention Test after Lesson #7** (without game pieces)
84.2% (m=5.05)	**81%** (m=4.86)

t(P6, P7-R3) = 1.83, not significant

Finally, the students participating in the study had increased their test score from 26.2% on the pre-test to a retention-test score of 81%. This gain was very large and statistically significant and shows that following a three-week period without any HOE instruction, the students were able to use the pictorial notation to demonstrate a significant gain in their ability to work with the equations presented in this study.

Grade 4, n=111
Study #131MA

Pre-test	**3-Week Retention Test after Lesson#7** (without game pieces)
26.8% (m=1.61)	**81%** (m= 4.86)

t(P, P7) = 19.49, Sig @.01

Note: Appendix 6 will provide the percentage of students who obtained each individual item correct on each of the tests. For example, the percentage of regular 4th grade students who obtained the correct response to $4x + 3 = 3x + 9$ increased from 8% on the pre-test to 79% on the three-week retention test without the use of the game pieces.

BROWARD COUNTY FIFTH GRADE STUDY #138MA
Regular Students

(Please read the General Introduction on pages 1 - 7 prior to reading the section below.)

Three regular 5th grade classrooms, consisting of 84 students, were analyzed together in meta-analysis #138MA.

TEACHERS OF THE STUDY

The teachers participating in this study were selected from among those who responded to an invitation from the mathematics supervisor to take part in this research project. The invitation requested teachers who had never taught HOE, who did not have a class set of materials, and who would be willing to train other teachers in their school. Two of the teachers in this study had more than 10 years of teaching experience; one teacher had between 1–3 years. All three teachers stated that they taught the program as instructed and that they made no changes in the teaching procedures.

STUDENTS OF THE STUDY

The students in this study were 84 5th grade students, 80% of whom were described by their teacher as inner-city students; the rest were described as suburban. Of the students in this group, one was described as LD, two as ELL, and one as GT.

CLASSES OF THE STUDY

Three classes were included in this study. Although summary forms for four classes were received, one of the classes could not be incorporated into this meta-analysis since the wrong test format was used. The result of this one class is nonetheless shown in Appendix 5. The results of another 5th grade class were lost in transit from the teacher to the supervisor, and never received by the researchers. The results of that class also were not included in this study.

CLASSROOM INSTRUCTION USED IN THE STUDY

The teachers were to present each of the first seven lessons of the HOE program as instructed in the training seminar. Each lesson involved the teacher presenting a concept to the class, along with two or three practice examples. This instructional component varied among the classes, which one teacher requiring about 20 minutes per lesson, another 30 minutes and another one hour. The average for the lesson presentation for this group of three teachers was about 35 minutes. Following this learning session, the students were provided with a worksheet to complete. Four of the examples on the worksheet were on the current lesson; six of the examples reviewed concepts learned in prior lessons. The worksheets were specifically designed in this manner so that the students would be reviewing all prior lessons each time they did a worksheet. The time spent on the worksheets varied from 20 minutes to 40 minutes. The average time spent on the worksheets for these three classes was about 30 minutes.

For the first six lessons of the program, the teacher used the Teacher's Demonstration Scale and Teacher Game Pieces to illustrate the equations and concepts. The students used their sets of game pieces and their flat laminated balance for these lessons. For Lesson #7, the teacher illustrated the pictorial solutions on the blackboard, and the students presented their solutions on

paper at their desks. The worksheet for Lesson #7 contained four pictorial examples and six review examples using the game pieces.

TESTING PROCEDURE USED IN THE STUDY

A pre-test was given to the students before they were exposed to the HOE program. At the conclusion of Lesson #6, the students were provided with a post-test in which they were at liberty to use their game pieces (the pawns, cubes, and laminated scale). The students were then taught Lesson #7, and given a second (different) post-test. This time the students were to take the post-test without using the game pieces. The students, however, were free to use the pictorial notation they had learned in Lesson #7. Finally, three weeks later, following a three-week period of no HOE instruction, the students were given a retention test (different from the two post-tests, but with similar questions). On this retention test the students were also not allowed to use the game pieces, but could use the pictorial notation learned in lesson #7.

All of the classes were taught by teachers who had participated in a one-day Making Algebra Child's Play workshop conducted by a certified Borenson and Associates instructor. The training of the teachers took place on October 25, 2007. The teachers began teaching HOE to their students almost immediately after the training (pre-test given to individual classrooms between October 29 and October 30, 2007). The first six lessons were taught and the Lesson #6 post-test with the game pieces was administered between November 5 and 8, 2007. The seventh lesson was then presented to the students and the Lesson #7 post-test was administered between November 6 and November 9, 2007. Three weeks later the retention test was administered (between November 30 and December 14, 2007).

This study is a meta-analysis of three separate classroom studies, each involving an intact classroom of 5th grade students. These classes consisted of almost all regular students. One class had one LD student; another class had one GT and two LD students. For the purposes this study, all the students were treated the same way both for the analysis of each individual class and for the combined group. Two of the classrooms were described by their teachers as inner-city schools and one was a suburban school.

STUDY HYPOTHESES: 5th GRADE STUDY #138MA, REGULAR STUDENTS

1. Students would score poorly on the pre-test. This result was expected since normally 5th graders (who do not have HOE) are not exposed to many of the algebraic equations presented on this pre-test. (See all test items in Appendix 3)
2. Students would score very well (in the 85% range) on the post-test after Lesson #6, using the game pieces. There would be a large and statistically significant increase from the pre-test to the post-test means after Lesson #6.
3. Students would score very well (in the 85% range) on the post-test after Lesson #7, in which the students do not use the game pieces, and there would be a large and statistically significant increase in test score means from the pre-test to the post-test after Lesson #7.
4. There would <u>not</u> be a statistically significant loss between the means on the post-test after Lesson #6 using the game pieces, and the post-test after Lesson #7, using the pictorial notation, if there was a loss at all. All prior studies showed either no difference or showed a significant increase in going to the pictorial notation.
5. Students would score lower on the three-week retention test than they did on the post-tests after Lesson #6 and Lesson #7, but we did not know if the loss would be significant.
6. Students would score well enough on the three-week retention test to produce a significant gain over the pre-test score.

RESULTS

The table below gives all the test results, with the t-scores noted beneath the table.

Regular Fifth Grade Classes (Study 138 MA)

	Pre-test	Post-test after Lesson #6	Post-test after Lesson #7	3-Week Retention Test after Lesson#7
Grade 5, n=84 **Study #138MA**	**37.7%** **(m=2.26)**	**88.3%** **(m=5.30)**	**88.5%** **(m=5.31)**	**84.7%** **(m= 5.08)**

t(P, P6) = 19.62 t(P, P7) = 17.09 t(P, P7-R3) = 14.71

t(P6, P7)=.11 t(P7, P7-R3) = 1.73 (P6, P7-R3) = 1.48

We note the following results:

a) The pre-test score was low as expected, since most of the equations presented in this study are not normally part of the 5th grade curriculum. We note that the pre-test score for the 5th graders (37.7%) was higher than for the 4th graders (26.8%).
b) The gain from the pre-test to each of the post-tests, one with the game pieces, and one without, was large and highly significant
c) There was no loss of achievement in going from the Lesson #6 post-test using the game pieces to the Lesson #7 post-test using the pictorial notation (t=.11)
d) The difference between the Lesson #7 post-test and the three week-retention test, both conducted with the pictorial notation, was not significant (t=1.73)
e) The difference between the Lesson #6 post-test, conducted with the game pieces, and the three week-retention test, conducted with the pictorial notation, was not significant (t=.1.48)
f) The gain in score from the pre-test (37.7%) to the retention test (84.7%), which did not use the game pieces, was very large and highly significant, with a t-value of 14.71.

BROWARD COUNTY FIFTH GRADE STUDY REGULAR STUDENTS #138MA

SUMMARY

This study demonstrated that the combined group of 84 5th grade students, more than 80% of whom were described by their teachers as inner city students, a) achieved a large and highly significant gain from the pre-test to the post-test following Lesson #6 in which the students used the game pieces, and b) that this significant gain was maintained on the post-test following Lesson #7, where the students did <u>not</u> use the game pieces (rather, they used the pictorial notation learned in Lesson #7).

Additionally, the students maintained their achievement level on a retention test administered three weeks later, without the use of the game pieces, and with no HOE instruction in the interim. The gain from the pre-test average of 37.7% to the retention test average of 84.7% was very large and highly significant.

<u>Note</u>: Appendix 6 will provide the percentage of students who obtained each individual item correct on each of the tests. For example, the percentage of regular 5th grade students who obtained the correct response to $4\boldsymbol{x} + 3 = 3\boldsymbol{x} + 9$ increased from 10% on the pre-test to 87% on the three-week retention test without the use of the game pieces

BROWARD COUNTY FIFTH GRADE STUDY
High Achieving/Gifted Students

(Please read the General Introduction on pages 1 - 7 prior to reading the section below.)

Five gifted, high-achieving 5th grade classrooms, consisting of 111 students, were analyzed together in meta-analysis #139MA.

TEACHERS OF THE STUDY

The teachers participating in this study were selected from among those who responded to an invitation from the mathematics supervisor to take part in this research project. The invitation requested teachers who had never taught HOE, who did not have a class set of materials, and who would be willing to train other teachers in their school. Three of the teachers in this study had 5 – 10 years of teaching experience; one had less than 3 years, and one had more than 10 years. All five teachers stated that they taught the program as instructed and that they made no changes in the teaching procedures.

STUDENTS OF THE STUDY

The students in this study were 111 5th grade students, about 33% of whom were described by their teachers as being inner city students and 66% as being urban students. Only one of the students had been exposed to HOE prior to this study.

The reports submitted by the teachers indicates that of the five classes one consisted entirely of gifted students, one had 75% gifted, and one had 25% gifted. The other two classes listed their students as regular students. However, when all of these five classes achieved unusually high pre-test scores, an inquiry was sent to the teachers questioning the high pre-test scores, and inquiring if any HOE instruction had taken place prior to the study. The response was received that all of these classes consisted of gifted and/or high achieving students, and that the pre-tests had been administered prior to beginning the HOE lessons.

CLASSES OF THE STUDY

Five summary forms were received, and each met the condition required to be included in the study. All the classes used the appropriate tests at the appropriate time.

CLASSROOM INSTRUCTION USED IN THE STUDY

The teachers were to present each of the first seven lessons of the HOE program as instructed in the training seminar. Each lesson involved the teacher presenting a concept to the class, along with two or three practice examples. This instructional component varied among the classes from 15 minutes to 35 minutes. The average for the lesson presentation for this group of five teachers was about 24 minutes. Following this learning session, the students were provided with a worksheet to complete. Four of the examples on the worksheet were on the current lesson; six of the examples reviewed concepts learned in prior lessons. The worksheets were specifically designed in this manner so that the students would be reviewing all prior lessons each time they did a worksheet. The time spent on the worksheets varied from 10 minutes to 30 minutes. The average time spent on the worksheets for these three classes was about 17 minutes.

For the first six lessons of the program, the teacher used the Teacher's Demonstration Scale and Teacher Game Pieces to illustrate the equations and concepts. The students used their sets of

game pieces and their flat laminated balance for these lessons. For Lesson #7, the teacher illustrated the pictorial solutions on the blackboard, and the students presented their solutions on paper at their desks. The worksheet for Lesson #7 contained four pictorial examples and six review examples using the game pieces.

TESTING PROCEDURE USED IN THE STUDY

A pre-test was given to the students before they were exposed to the HOE program. At the conclusion of Lesson #6, the students were provided with a post-test in which they were at liberty to use their game pieces (the pawns, cubes, and laminated scale). The students were then taught Lesson #7, and given a second (different) post-test. This time the students were to take the post-test without using the game pieces. The students, however, were free to use the pictorial notation they had learned in Lesson #7. Finally, three weeks later, following a three-week period of no HOE instruction, the students were given a retention test (different from the two post-tests, but with similar questions). On this retention test the students were also not allowed to use the game pieces, but could use the pictorial notation learned in Lesson #7.

All of the classes were taught by teachers who had participated in a one-day Making Algebra Child's Play workshop conducted by a certified Borenson and Associates instructor. The training of the teachers took place on October 25, 2007. The teachers began teaching HOE to their students almost immediately after the training (pre-test given to individual classrooms between October 29 and October 31, 2007, with the exception of one teacher who provided the pre-test on November 12th). The first six lessons were taught and the Lesson #6 post-test with the game pieces was administered between November 5 and 13, 2007, with the exception of the one teacher who provided it on November 28, 2007. The seventh lesson was then presented to the students and the Lesson #7 post-test was administered between November 6 and November 30, 2007. Three weeks later the retention test was administered (between November 30 and December 21, 2007).

This study is a meta-analysis of five separate classroom studies, each involving an intact classroom of 5th grade gifted or highly talented students. Three of the classrooms were suburban, one was inner-city, and the other was a mixture of both.

STUDY HYPOTHESES: 5th GRADE STUDY #139MA, HIGH ACHIEVING/GIFTED STUDENTS

1. We expected 5th grade gifted students to better on the pre-test than regular 5th grade students, but we expected scores below 50%. This result was expected since normally 5th grade gifted students (who do not have HOE) are not exposed to many of the algebraic equations presented on this pre-test. (See all test items in Appendix 3)
2. Students would score very well (in the 90% range) on the post-test after Lesson #6, using the game pieces. There would be a large and statistically significant increase from the pre-test to the post-test means after lesson #6.
3. Students would score very well (in the 90% range) on the post-test after Lesson #7, in which the students do not use the game pieces, and there would be a large and statistically significant increase in test score means from the pre-test to the post-test after Lesson #7.
4. There would <u>not</u> be a statistically significant loss between the means on the post-test after Lesson #6 using the game pieces, and the post-test after Lesson #7, using the pictorial notation, if there was a loss at all. All prior studies showed either no difference or showed a significant increase in going to the pictorial notation.

5. Students would score lower on the three-week retention test than they did on the post-tests after Lesson #6 and Lesson #7, but we did not know if the loss would be significant.
6. Students would score well enough on the three-week retention test to produce a significant gain over the pre-test score.

RESULTS
The table below gives all the test results, with the t-scores noted beneath the table.

Gifted or High Achieving Fifth Grade Classes (Study 139 MA)

	Pre-test	**Post-test after Lesson #6**	**Post-test after Lesson #7**	**3-Week Retention Test after Lesson#7**
Grade 5, n=111 **Study #139MA**	**78%** **(m=4.68)**	**95.3%** **(m=5.72)**	**95.3%** **(m=5.72)**	**94.2%** **(m= 5.65)**

t(P, P6) = 8.06 t(P, P7) = 8.14 t(P, P7-R3) = 6.05
t(P6, P7)=0.0 t(P7, P7-R3) = .95 (P6, P7-R3) = .90

a) The pre-test score of 78% was much higher than expected. It was double the pre-test score of the regular 5th grade group (37.7%)
b) The gain from the pre-test to each of the post-tests, one with the game pieces, and one without, was significant
c) There was no loss of achievement in going from the Lesson #6 post-test using the game pieces to the Lesson #7 post-test using the pictorial notation
d) The difference between the Lesson #7 post-test and the three week-retention test, both conducted with the pictorial notation, was not significant (t=.95)
e) The difference between the Lesson #6 post-test, using the game pieces, and the three week-retention test, without the game pieces, was not significant (t=.90)
f) The gain in score from the pre-test (78%) to the retention test (94.2%) was significant, with a t-value of 6.05

BROWARD COUNTY 5th GRADE STUDY: GIFTED AND TALENTED STUDENTS #139MA
SUMMARY

This study demonstrated that the combined group of 111 5th grade gifted and or high-achieving students, a third of whom were described by their teachers as inner city students, a) achieved a significant gain from the pre-test to the post-test following Lesson #6 in which the students used the game pieces, and b) that this significant gain was maintained on the post-test following Lesson #7, where the students did <u>not</u> use the game pieces (rather, they used the pictorial notation learned in Lesson #7).

Additionally, the students maintained their achievement level on a retention test administered three weeks later, without the use of the game pieces, and with no HOE instruction in the interim. The gain from the pre-test average of 78% to the retention test average of 94.2% was significant. Even though these 5th graders had an unusually high pre-test score, they were nonetheless able to obtain a significant statistical increase in score through the use of the program.

<u>Note</u>: Appendix 6 will provide the percentage of students who obtained each individual item correct on each of the tests. For example, the percentage of students who obtained the correct response to $4x + 3 = 3x + 9$ increased from 71% on the pre-test to 94% on the three-week retention test without the use of the game pieces

GENERAL SUMMARY
BROWARD COUNTY RESEARCH STUDY

A total of 326 students from 14 different classes were included in this study. The raw scores and percentage scores are shown below. We note that the average 4th graders saw their scores triple from the pre-test to each of the post-tests and to the retention test; the average 5th graders saw their scores more than double from the pre-test to these post-tests and to the retention test.

	Pre-test	Post-test after Lesson #6	Post-test after Lesson #7	3-Week Retention Test after Lesson#7
Grade 4, n=111 **Study #131MA** **Regular students**	**26.8%** **(m=1.61)**	**84.2%** **(m=5.05)** **t(P, P6)=20.50**	**84.2%** **(m=5.05)** **t(P, P7)=20.45**	**81%** **(m= 4.86)** **t(P, P7-R3)=19.49**
Grade 5, n=84 **Study #138MA** **Regular students**	**37.7%** **(m=2.26)**	**88.3%** **(m=5.30)** **t(P, P6)= 19.62**	**88.5%** **(m=5.31)** **t(P, P7)=17.09**	**84.7%** **(m= 5.08)** **t(P, P7-R3)=14.71**
Grade 5, n=111 **Study #139MA** **Gifted/Talented**	**78%** **(m=4.68)**	**95.3%** **(m=5.72)** **t(P, P6)=8.06**	**95.3%** **(m=5.72)** **t(P, P7)=8.14**	**94.2%** **(m= 5.65)** **t(P, P7-R3)=6.05**

These three meta-analyses demonstrate that 1) Each of the combined group of 111 regular 4th graders, 84 regular 5th graders, and 111 gifted and talented 5th graders achieved a large and significant gain from the pre-test to the post-test following Lesson #6, and 2) This significant gain was maintained on the post-test following Lesson #7, where the students did not use the game pieces (rather, they used the pictorial notation learned in Lesson #7). These results confirm the results of previous studies conducted with 4th, 6th and 8th graders that students who learn the Hands-On Equations (HOE) methods of solving equations can be equally successful with or without the game pieces. In other words, the students are able to transfer their hands-on learning to the pictorial method presented in Lesson #7, which uses only paper and pencil, and be equally successful in solving the equations.

Additionally, the current study showed that after a three-week period of no HOE instruction, the students performed essentially the same as they did three weeks earlier on the Lesson #6 and Lesson #7 post-tests. Since the three-week retention test was conducted without the use of the game pieces, the current study demonstrates that 4th and 5th grade students are able to retain the methods they have learned in the program and are able to solve algebraic equations using the pictorial notation even after a period of three weeks without HOE instruction.

In summary, the results obtained in this study are consistent with previous studies which show that when teachers who have been trained in the Hands-On Equations program instruct their students in the use of the program, and go through the first seven lessons of the program as prescribed, the students learn the algebraic concepts presented, they do well on the posts-tests, and they remember what they learn, with or without the use of the game pieces.

The Authors of the Study

Larry W. Barber has served as past vice president of the American Educational Research Association. He has served as an assistant superintendent of schools and for 19 years was Director of Research for Phi Delta Kappa.

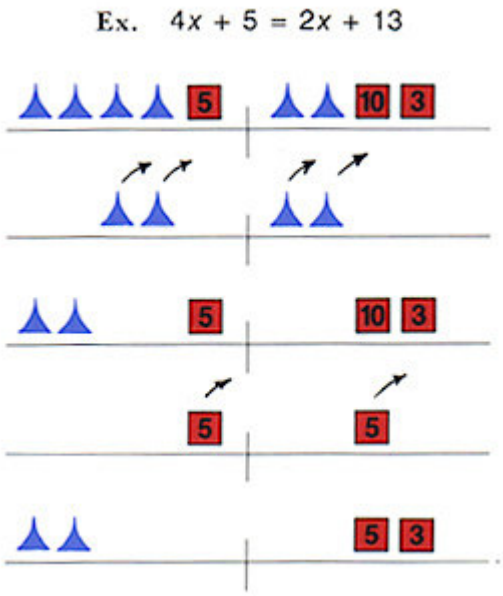
Henry Borenson received his doctorate in educational administration from Teachers College, Columbia University. His teaching experience includes teaching students in the South Bronx of New York City as well as students who represented the U.S. in the International Mathematical Olympiad while a teacher at Stuyvesant High School. Borenson received a U.S. patent for the Hands-On Equations teaching methodology. He is currently President of Borenson and Associates, Inc. Since 1990, more than 25,000 teachers have attended the Making Algebra Child's Play Workshop which he designed to help teachers obtain the maximum value from Hands-On Equations.

References

1) Barber, L.W., Forbes, R.H., and Fortune, J.C. Developing Research Skills for Professional Educators (Participants Manual). Bloomington, IN: Phi Delta Kappa, 1988.
2) Barber, L.W., and Langdon, C., Academic Underachievement in the First Grade: Final Report of the Joint Study by Phi Delta Kappa and the Russian Academy of Education: Bloomington, IN: Phi Delta Kappa, 1994.
3) Barber, L.W., Carbo, M., and Thomasson, R., A Comparative Study of the Reading Styles Program to Extant Programs of Teaching Reading, Bloomington IN: Phi Delta Kappa, 1998.
4) Barber, L.W. and Borenson, H., Hands-On Equations Interim Results, July 7, 2007 (Report #33c): The Effect of Hands-On Equations on the Learning of Algebra by 5th Grade Inner City Students, Borenson and Associates, Inc. 2007
5) Barber, L.W. and Borenson, H. Hands-On Equations, Interim Report, Nov 19, 2007: A Comparison of Algebra Achievement by 4th, 6th and 8th graders, Borenson and Associates Inc, 2007.
6) Barclay, J., A study of a Manipulative Approach to Teaching Linear Equations to Sixth Grade Students. (Masters Degree Dissertation), Texas Woman's University, Denton Texas, 1992.
7) Borenson, Henry. The Hands-On Equations Learning System. Borenson and Associates, Allentown, PA. 1986.
8) Dembo, Myron H., and Gibson, Sherry, Teacher's Sense of Efficacy: An Important Factor in School Improvement, The Elementary School Journal, Vol. 86, No. 2 (Nov. 1995), p. 173)
9) Edwards, A.L. Statistical Methods for the Behavioral Sciences, Holt Rinehart, and Winston, New York, 1963.
10) Frymier, J., Barber, L.W., et al Manual of Instruction for A Study of Students at Risk. Bloomington IN: Phi Delta Kappa, 1988.
11) Frymier, J., Barber L.W., et al "Simultaneous Replication: A Technique for Large Scale Research." Phi Delta Kappa, vol. 71, no. 3, November 1989, pp. 228-231.
12) Frymier, J., Robertson, N. and Barber, L.W., Learning to Fail: Case Studies of Students at Risk, 129 pages. Bloomington, IN. Phi Delta Kappa, 1991.
13) Frymier, J., Barber, L.W., et al., Growing up is Risky Business and Schools are not to Blame. 240 pages. Bloomington, IN: Phi Delta Kappa, 1992 a.
14) Frymier, J., Barber, L.W., et al., Assessing and Predicting Risk among Students in School, 351 pages. Bloomington. IN: Phi Delta Kappa, 1992 b.
15) Kearns, Sophie, The Relationship Between Teacher Efficacy, Educational Beliefs and Optimism Bias Among Preservice Teacher, paper presented at the Australian Association for Research in Education, 1995 Conference.
16) Slavin R.E., & Madden, N.A., (1993, April). Multi-site replicated experiments: An application to success for all. Paper presented at the annual meeting of the American Educational Research Association, Atlanta.

Appendix 1

The essential method used in Hands-On Equations is illustrated by the following example:



***x*=4**

The *x*'s are represented by blue pawns and the constants by number cubes; the equation is "physically set" up on the laminated balance scale.

"Legal moves" are used to simplify the equation. In the 2nd step above, two blue pawns are removed from each side of the balance. The 3rd step indicates what is left. The 4th step shows the legal move of removing a 5 value from the cubes on both sides. In the last step the student recognizes that each pawn has a value of 4.

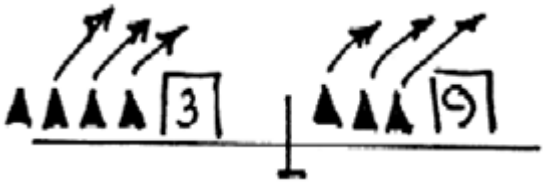
Appendix 2
HANDS-ON EQUATIONS® LEARNING SYTEM
Teaching Objectives

Equation	Teaching Point/Objective
Level I, Lessons 1 – 7	
Lesson 1 	In any specific problem, all the blue pawns have the same value, and the scale is in balance. **Solve for the pawn using *trial and error* and intuitive thinking**.
Lesson 2 $2x + x = x + 8$	The pawn has a special name, "***x***". **Transform the equation into its physical representation**, using the blue pawn for the ***x*** and the red number cube for the number constant. The two sides of the equal sign become the two sides of the scale. Use trial and error to find the value of x.
Lesson 3 $4x + 2 = 3x + 9$	The *legal move* with pawns is introduced: **we may remove the same number of blue pawns from both sides of a balanced system** (Subtraction Property of Equality).
Lesson 4 $4x + 5 = 2x + 13$	The legal move with the cubes is introduced: **We may subtract the same number cube or cube value from both sides of a balanced system** (Subtraction Property of Equality).
Lesson 5 $5x - 3x + 2 = x + 5$	**Students take away pawns as part of the setup process.** Distinguish the set up from a legal move, which comes after the setup has been completed.
Lesson 6 $2(x + 3) = x + 8$	**The students learn that the number outside the parenthesis indicates how many times the expression inside the parenthesis is set up on the balance scale.** (Some students learn the distributive law without being taught!)
Lesson 7 $4x + 3 = 3x + 9$	**Transfer the hands-on experiences of Level I to a pictorial system.** The ***x***'s are represented by shaded triangles, the number constants by boxed numbers, and the balance scale by a drawing of the scale. No plus signs are placed on the scale, only pawns or cubes.

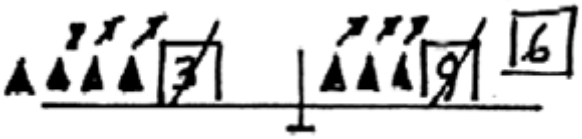
Appendix 3

PICTORIAL NOTATION
$4x + 3 = 3x + 9$

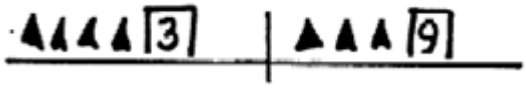
The *x's* are represented by shaded triangles; the constants by boxed numbers. The equality of the two sides is indicated by the two sides of the balance scale.



Legal moves may be illustrated by erasing, crossing out, or using arrows. In the above example, most students will see that the pawn is worth 6. If the student wishes, he/she may cross off the 3 cube and replace the 9 cube with a 6 cube:



It is now clear that the pawn is worth 6. In order to conduct the check, the student redraws the original setup:



We see that the check is: 27 = 27

Appendix 4
TEST QUESTIONS FOR STUDY #131MA

Pre-Test Questions	**Post-Test After Lesson #7**
1. $2x = 8$	**1. $2x = 6$**
2. $x + 3 = 8$	**2. $x + 3 = 10$**
3. $2x + 1 = 13$	**3. $2x + 1 = 7$**
4. $3x = x + 12$	**4. $3x = x + 2$**
5. $4x + 3 = 3x + 6$	**5. $4x + 3 = 3x + 7$**
6. $2(2x + 1) = 2x + 6$	**6. $2(2x + 1) = 2x + 10$**
Post -Test After Lesson #6	**Three-Week Retention Test**
1. $2x = 10$	**1. $2x = 4$**
2. $x + 3 = 8$	**2. $x + 3 = 13$**
3. $2x + 2 = 10$	**3. $2x + 1 = 17$**
4. $3x = x + 4$	**4. $3x = x + 14$**
5. $4x + 3 = 3x + 9$	**5. $4x + 3 = 3x + 8$**
6. $2(2x + 1) = 2x + 8$	**6. $2(2x + 3) = 2x + 10$**

Appendix 5
Studies not Included in the Broward County Meta Analyses

Each of the studies noted below had a technical fault that required us to exclude it from the meta-analyses. In some cases, the study was missing the retention test; in other cases the wrong test series was used*. In one case, the retention test score was much larger than the post-test score. Although these studies could not be used in the meta-analyses, we report their scores below.

6-question format (retention test much higher than post-test)

Class Code	Pre-test	Post-test after Lesson #6	Post-test after Lesson #7	3-Week Retention Test after Lesson #7
C1-4th grade	**25.7%** **(m=1.54)**	**59%** **(m=3.54)**	**69.5%** **(m=4.17)**	**81.3%** **(m= 4.88)**

8-question format

Class Code	Pre-test	Post-test Lesson #6	Post-test Lesson #7
C2-4th grade	**56.3%** **m=4.50**	**81.3%** **m=6.50**	**97.2%** **m=7.78**

8-question format

Class Code	Pre-test	Post-test Lesson #6	Post-test Lesson #7
C3-4th grade	**33.3%** **m=2.67**	**69.6%** **m=5.59**	**66.6%** **m=5.33**

6-question format (no 3-week retention test)

Class Code	Pre-test	Post-test Lesson #6	Post-test Lesson #7
C4-4th grade	**37%** **m=2.22**	**56.3%** **m=3.38**	**73%** **m=4.38**

8-question format

Class Code	Pre-test	Post-test Lesson #6	Post-test Lesson #7
C5-5th grade	**33.1%** **m=2.65**	**91.1%** **m=7.29**	**91.1%** **m=7.29**

C6: 5th grade study lost within district; not submitted to the researchers.

*The researchers take responsibility for this error. Each class set came with a version of the tests different from the ones that were to be used in this study. We did not caution the teacher to use only the version distributed at the training.

Appendix 6

Meta Analysis # 131 MA (4th Grade Broward Regular)
Test Item Analysis by Test Item Number

Pre-test	Students	#1	#2	#3	#4	#5	#6
125	17	12	11	3	1	2	0
127	21	13	20	9	5	2	1
128	16	6	16	6	1	1	1
129	23	8	23	1	0	1	2
130	17	9	8	5	0	0	1
131	17	5	0	0	3	3	4
Totals	111	53	78	24	10	9	9
%	---	0.48	0.70	0.22	0.09	0.08	0.08

Post-test 6	Students	#1	#2	#3	#4	#5	#6
125	17	17	15	14	17	12	9
127	21	21	21	20	21	20	19
128	16	16	16	16	16	16	15
129	23	23	20	22	23	20	19
130	17	15	14	13	13	11	7
131	17	10	14	10	11	10	10
Totals	111	102	100	95	101	89	79
%	---	0.92	0.90	0.86	0.91	0.80	0.71

Post-test 7	Students	#1	#2	#3	#4	#5	#6
125	17	16	16	16	16	13	8
127	21	21	21	20	21	20	16
128	16	16	16	16	16	16	16
129	23	23	23	23	21	22	18
130	17	15	16	13	13	7	10
131	17	12	14	10	11	11	6
Totals	111	103	106	98	98	89	74
%	---	0.93	0.95	0.88	0.88	0.80	0.67

Retention	Students	#1	#2	#3	#4	#5	#6
125	17	16	15	14	15	11	4
127	21	21	20	18	18	19	12
128	16	16	16	15	16	16	14
129	23	22	22	21	22	22	17
130	17	16	13	13	11	10	9
131	17	11	13	10	13	10	9
Totals	111	102	99	91	95	88	65
%	---	0.92	0.89	0.82	0.86	0.79	0.59

Appendix 7

Meta Analysis # 138 MA (5th Grade Broward Regular)

Test Item Analysis by Test Item Number

Pre-test	**Students**	**#1**	**#2**	**#3**	**#4**	**#5**	**#6**
136	48	33	39	13	4	3	1
137	19	17	18	15	2	2	0
138	17	7	16	7	7	3	3
Totals	84	57	73	35	13	8	4
%	---	0.68	0.87	0.42	0.15	0.10	0.05

Post-test 6	**Students**	**#1**	**#2**	**#3**	**#4**	**#5**	**#6**
136	48	46	44	42	44	34	31
137	19	19	18	18	18	17	14
138	17	17	17	17	16	16	17
Totals	84	82	79	77	78	67	62
%	---	0.98	0.94	0.92	0.93	0.80	0.74

Post-test 7	**Students**	**#1**	**#2**	**#3**	**#4**	**#5**	**#6**
136	48	45	44	42	37	41	39
137	19	19	17	18	19	14	14
138	17	17	17	16	17	17	14
Totals	84	81	78	76	73	72	67
%	---	0.96	0.93	0.90	0.87	0.86	0.80

Retention	**Students**	**#1**	**#2**	**#3**	**#4**	**#5**	**#6**
136	48	45	46	38	43	42	43
137	19	18	17	15	16	15	9
138	17	16	15	17	16	16	10
Totals	84	79	78	70	75	73	62
%	---	0.94	0.93	0.83	.89	0.87	0.74

Appendix 8

Meta Analysis # 139 MA (5th Grade Broward Gifted)
Test Item Analysis by Test Item Number

Pre-test	Students	#1	#2	#3	#4	#5	#6
132	18	17	18	16	12	11	3
133	20	19	19	16	14	17	16
134	25	25	25	25	18	21	10
135	20	19	20	19	14	15	10
139	28	28	24	21	21	15	12
Totals	111	108	106	97	79	79	51
%	---	0.97	0.95	0.87	0.71	0.71	0.46

Post-test 6	Students	#1	#2	#3	#4	#5	#6
132	18	18	18	16	17	18	16
133	20	20	20	20	20	17	17
134	25	25	25	25	25	25	25
135	20	20	20	20	19	19	18
139	28	28	28	26	26	24	20
Totals	111	111	111	107	107	103	96
%	---	1.00	1.00	0.96	0.96	0.93	0.86

Post-test 7	Students	#1	#2	#3	#4	#5	#6
132	18	18	17	16	17	17	15
133	20	20	20	20	20	17	18
134	25	25	25	24	25	25	22
135	20	20	20	20	20	20	18
139	28	28	28	26	26	25	23
Totals	111	111	110	106	108	104	96
%	---		0.99	0.95	0.97	0.94	0.86

Retention	Students	#1	#2	#3	#4	#5	#6
132	18	17	17	18	17	18	14
133	20	20	18	19	16	19	18
134	25	25	24	25	24	24	23
135	20	20	20	20	19	19	18
139	28	28	28	28	25	25	21
Totals	111	110	107	110	101	105	94
%	---	0.99	0.96	0.99	0.91	0.95	0.85

Appendix 9
Item Analysis

Below, we show the percentage of students who obtained the item correct on the pre-test vs. the percentage of students who obtained the comparable item correct on the three-week retention test for each of the three meta-analyses.

Grade 4, n =111. Study #131MA Regular Students
Percentage of Students with Correct Item Response

	Equation	Pre-test	Retention-test
Question #1	$2x = 8$	48%	92%
Question #2	$x + 3 = 8$	70%	89%
Question #3	$2x + 1 = 13$	22%	82%
Question #4	$3x = x + 12$	9%	86%
Question #5	$4x + 3 = 3x + 6$	8%	79%
Question #6	$2(2x+1) = 2x + 6$	8%	59%

Grade 5, n =84. Study #138MA Regular Students
Percentage of Students with Correct Item Response

	Equation	Pre-test	Retention-test
Question #1	$2x = 8$	68%	94%
Question #2	$x + 3 = 8$	87%	93%
Question #3	$2x + 1 = 13$	42%	83%
Question #4	$3x = x + 12$	15%	89%
Question #5	$4x + 3 = 3x + 6$	10%	87%
Question #6	$2(2x+1) = 2x + 6$	5%	74%

Grade 5, n =111. Study #139MA Gifted/Talented Students
Percentage of Students with Correct Item Response

	Equation	Pre-test	Retention-test
Question #1	$2x = 8$	97%	99%
Question #2	$x + 3 = 8$	95%	96%
Question #3	$2x + 1 = 13$	87%	99%
Question #4	$3x = x + 12$	71%	91%
Question #5	$4x + 3 = 3x + 6$	71%	95%
Question #6	$2(2x+1) = 2x + 6$	46%	85%