Hands-on Equations Research, Interim Results: Study #59a, August 23, 2007
The Effects of Hands-on Equations® on the Learning of Algebra by 4th Grade Students: A Comparison of Achievement with and Without the Game Pieces

Hands-On Equations® (HOE), developed by Dr. Henry Borenson, uses numbered-cubes to represent the constants, and blue pawns to represent the variable x. It also uses a scale representation on which the students "set up" the equation. The students then proceed to use "legal moves," which are the mathematical counterpart of the abstract algebraic methods which are used to solve these linear equations. The system thus makes abstract linear equations visual and understandable, and further provides students with the means of solution through a kinesthetic approach which makes sense to them.

The program is unique in that the abstract knowledge base needed by students to solve these equations is transformed into an easily understood and manageable set of verbal, visual and kinesthetic responses using manipulatives. The program teaches algebraic principles which students in grade 3 to 8 can apply in any sequence desired to solve the given equation. Hence, the students using Hands on Equations need not memorize a series of steps to solve an equation, as is the case in more traditional instruction. Rather they feel empowered to use their thinking and understanding of basic principles to solve the problem at hand.

The research study uses a Multi-Site Replications Design and a Meta Analysis procedure to study the effect of the HOE program on many groups of students with different characteristics (regular education, special ed., gifted, elementary, middle-school, high school, etc). All of these groups of students will be studied separately. Presently we have data on more than 75 classrooms,

This particular study was designed to measure the effect of the first 7 lessons of the HOE program on the learning of algebra by 4th grade students in a large urban school district that included both inner city and suburban schools. In addition this study was designed to determine if there was any difference (large enough to be significant) in student achievement in learning HOE by having the students take a post-test using the HOE game pieces and having them take a post-test without the game pieces.

A pre-test was given to the students before they were exposed to the HOE program. At the conclusion of Lesson #6, the students were provided with a post-test in which they were at liberty to use their game pieces (the pawns, cubes, and laminated scale). The students were then taught Lesson #7, and given a second (different) post-test. This time the students were to take the post-test without using the game pieces. The students, however, were free to use the pictorial notation they had learned in Lesson #7.

In this study five of the six teachers had at least 5 years of teaching experience; however one teacher was in the first year of teaching. For all the teachers this was their first experience teaching HOE. All the teachers were trained by a Borenson and Associates, Inc. instructor on April 30th, 2007 in a one day workshop. Each teacher started teaching their students in HOE procedures on a different day, between May 2, and May 10, 2007. They taught the first six lessons in approximately 7 days. Each teacher administered the post-test after Lesson #6 either immediately at the conclusion of the lesson, later that same day, or the next day. The teachers then taught Lesson #7 one or two days later and administered the post-test after Lesson #7 immediately at the conclusion of Lesson #7, later that day or the next day. The students were provided 15 minutes for the pre-test and each of the post-tests.

This study is a meta-analysis of 6 separate classroom studies, each involving an intact classroom of 4th grade students. Most of the classes had at least one or two LD students. One class listed 29% of the students as LD; another class listed 45% of their students as LD. For the purposes this study, all the students were treated the same way, whether classified LD or not, both for the analysis of each individual class and for the combined group. Three of the classrooms were in inner-city schools and three were in suburban schools, all part of the same large school district located in the southeastern United States.

RESULTS

Six classrooms were included in this meta-analysis (combined N = 123). Each classroom's data was analyzed independently to provide feedback to each teacher about the performance of their students. t tests were conducted between the means of the pre-test and the post-test after Lesson #6, and between the means of the pre-test and post-test after Lesson #7, and then between the means of the Lesson #6 and Lesson #7 post-tests. For each of the six individual classes in this study, the gain from the pre-test to each of the post-tests was statistically significant. In four of the classes the t-value obtained was more than 10.0 with at least a doubling of the score from the pre-test to each of the post-tests.

For the combined group of the six classes, the effect sizes between the pre-test and post-test after Lesson #6, and between the pre-test and post-test after Lesson #7 were large and highly significant. The gain between the pre-test mean (1.81) and the post-test mean after Lesson #6 (5.04) produced a t-value of 22.62; the gain between the pre-test mean (1.81) and post-test after Lesson #7 mean (5.32) produced a t-value of 29.70.

For the combined group of the six classes, the effect size between the post-test mean (5.04) after Lesson #6 and the post-test mean (5.32) after Lesson #7 was significant but small (t=2.86). Only one of the six classes (Study #54) when analyzed alone (for the difference between the post-test after Lesson #6 and the post-test after Lesson #7) had a t-value that was large enough to be significant (t=3.56). Since this significance was only observed in one of the individual class studies, and since the increase in scores is relatively small (from 84% to 88.7%) we are not prepared to assert any significant gain by the students in using the pictorial notation over using the game pieces.

CONCLUSIONS:

This study demonstrated that the combined group of 123 4th grade inner city and suburban school students a) achieved a large and highly significant gain* from the pre-test to the post-test following Lesson #6, and b) that this significant gain was maintained on the post-test following Lesson #7, where the students did <u>not</u> use the game pieces (rather, they used the pictorial notation learned in Lesson #7). This result, as well as that of Study #102b**, involving 190 6th grade students from a rural school district in the northwestern United States, demonstrates that students who learn the HOE methods of solving equations are able to be equally successful with or without the use of the game pieces. In other words, the students are able to transfer their hands-on learning to the pictorial method presented in Lesson #7, which uses only paper and pencil, and be equally successful in solving the equations.

* In percentage terms, the average pre-test score for the entire group of 123 4th grade inner city/suburban school students was 30.2%. The average score for the entire group on the Lesson #6 post-test <u>using</u> the game pieces was 84%. The average score for the entire group on the Lesson #7 post-test <u>without</u> using the game pieces was 88.7%. Copies of the tests are shown below.

** In percentage terms, the average pre-test score for the entire group of 190 6th grade rural school students was 48.2%. The average score for the entire group on the Lesson #6 post-test <u>using</u> the game pieces was 92.3%. The average score for the entire group on the Lesson #7 post-test <u>without</u> using the game pieces was 94%.

Report Submitted by Larry W. Barber, Ph.D., August 23, 2007

TEST QUESTIONS FOR STUDY #59a and 102b

Pre-Test Questions	Post-Test After Lesson #7
1. $2x = 8$	1. $2x = 6$
2. $x + 3 = 8$	2. $x + 3 = 10$
3. $2x + 1 = 13$	3. $2x + 1 = 7$
4. $3x = x + 12$	4. $3x = x + 2$
5. $4x + 3 = 3x + 6$	5. $4x + 3 = 3x + 7$
6. $2(2x + 1) = 2x + 6$	6. $2(2x + 1) = 2x + 10$
Post -Test after Lesson #6	
1. $2x = 10$	
2. $x + 3 = 8$	
3. $2x + 2 = 10$	
4. $3x = x + 4$	
5. $4x + 3 = 3x + 9$	
6. $2(2x + 1) = 2x + 8$	

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