Borenson Hands-On Equations Research Designs and Interim Results: December 2006
Effect of Making Algebra Child's Play® Seminar on
Teacher Self-Concept and Student Achievement

By Larry W. Barber and Henry Borenson

ABSTRACT

This is the first group of what are expected to be a number of research studies to determine the effectiveness of the Borenson **Making Algebra Child's Play** seminar upon teacher self-perception of their instructional ability to teach algebraic concepts to their lowest achieving students, with a major focus on teachers of grades 3 - 8. Additionally, this and following studies will attempt to measure the instructional effect of **Hands-On Equations®** upon students when taught by teachers who have attended one of these seminars. The current study analyzes the response of 739 teacher participants, as well as the pre to post test achievement of two classes, a 5^{th} grade class and a class of LD students in the 9^{th} and 11^{th} grade; it also reports on a 1992 study with 123 participants age 11 - 13.

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For 17 years Henry Borenson and his associates have been presenting to classroom teachers a system of instruction used to teach children, age 8 to 13, the basic concepts of algebra so that they are able to understand and solve algebraic linear equations. This system of instruction is called **Hands on Equations®.**

Hands-On Equations (HOE) uses numbered-cubes to represent the constants, and blue pawns to represent the variable ***x***. It also uses a scale representation on which the students "set up" the equation. The students then proceed to use "legal moves," which are the mathematical counterpart of the abstract algebraic methods which are used to solve these linear equations. The system thus makes abstract linear equations visual and understandable, and further provides students with the means of solution through a kinesthetic approach which makes sense to them.

The program is unique, in that the abstract knowledge base needed by students to solve these equations is transformed into an easily understood and manageable set of verbal, visual and kinesthetic responses using manipulatives. The program teaches algebraic principles which students in grades 3 to 8 can apply in any sequence desired to solve the given equation. **Hence, students using Hands-On Equations need not memorize a series of steps to solve an equation, as is the case in more traditional instruction. Rather, they feel empowered to use their thinking and understanding of basic principles to solve the problem at hand.**

As a part of the underlying set of principles that motivated the current study, the author assumes that teachers who believe that they can successfully teach algebraic concepts to even their "lowest-level" students are in fact more likely to be successful with all of their students than teachers who believe that they are not able to teach these concepts to their lowest-level students. Two assumptions are involved here. The first is that methods that are successful in teaching abstract concepts to the weakest or least successful students will also work with average and strong students. Secondly, there is the assumption that the teacher belief system makes a difference in student outcome. This assumption is borne out by studies on teacher efficacy and student achievement. For example, Dembo and Gibson note that comments made by teachers in conversations with colleagues, such as "If I just knew how to get through to Bill, I could help him," or "These kids just don't care about learning," carry significance that affect student learning: "Recent research indicates that such comments may be more than idle chatter; they may reflect important beliefs that influence teacher-student interactions and teachers' success in producing student achievement gains. Researchers have labeled these beliefs teachers' sense of efficacy—the extent to which teachers believe they can affect student learning" (Dembo and Gibson 1985).

Kearns (Kearns 1995) reported that "In the two decades since the concept of teacher efficacy was first introduced into the area of educational research studies have found evidence supporting the importance of the construct in an educational context. A significant relationship was discovered to exist between teacher efficacy and student achievement." She further adds, "**It has been determined that a teacher's sense of efficacy is perhaps the most significant predictor and contributor of teacher influence to student achievement** (Armor et al., 1976; Berman et al., 1977; Midgley et al., 1989)."

The current research design has in mind to measure teacher belief in their ability to successfully teach algebraic concepts to 80% or more of the students in their lowest-achieving class, both prior to and after treatment, and it also has in mind to measure teacher self-concept about their ability to teach algebraic concepts to their lowest-achieving students, both prior to and after treatment. The first measure within each line of inquiry is taken prior to the beginning of the seminar; the second measure is taken at the conclusion of the sixth lesson of the seminar, approximately 2½ hours into the full-day **Making Algebra Child's Play** seminar conducted by Borenson and Associates, Inc. The seminars were conducted by expert trainers who had conducted identical seminars over numerous years.

The **Making Algebra Child's Play** seminar usually begins with a live student demonstration. Several 4th or 5th grade students, who have not had previous experience with Hands-On Equations, are presented with the first three lessons of the program in a period of about 20 to 30 minutes. The students are seated at the front of the room, with the teacher participants behind them, and the instructor in front of them. The students respond to the Instructor from their seats and also come up to the front of the room to demonstrate what they are learning. The student demonstration* concludes once the students have physically represented and solved the equation $\mathbf{4x + 3 = 3x + 9}$ on the teacher's balance scale at the front of the room, and once they have verified that their solution works. After the students leave, the remainder of the seminar is interactive with the teacher participants. The teachers work with the manipulatives at their desks and also present their solutions at the front of the room. Generally, participants are provided with the opportunity to solve the new equation and to discover the new legal moves. The Instructor serves as a "coach" and will provide direct instruction when the participants themselves are not able to learn the new concept on their own. The Instructor will also clarify concepts, elaborate upon a solution being presented and comment on the pedagogy being used.

This study is being conducted in conjunction with other studies which set out to measure the extent of student acquisition of algebraic knowledge from teachers who have in fact been through the **Making Algebra Child's Play** seminar, or who have been instructed in the program by teachers who have attended such training, and who, as a result of the training, exhibit a high level of self-concept about their ability to teach these algebraic concepts to their lowest-achieving students. These are among the exploratory studies currently being undertaken to determine the major impact areas that the Hands-On Equations program has upon teachers and students. These exploratory studies will suggest other lines of investigation and will enable us to pursue more formal research in promising areas.

In medicine there is a long term research limitation for rare illnesses or rare diseases, where the population affected is too small to use statistical methods to determine the best treatment because the appropriate statistics are not available. In the past a particular medical school would adopt each disease and over time (decades) accumulate enough treatment data and consequent effect that the number grew large enough to rerun the data and learn enough about the disease and treatment to begin an effective regimen of treatment.

In education, Frymier, (Frymier, J., et al 1989) (Frymier, J., et al 1988) (Barber, L.W., et al, 1988) identified a similar common problem encountered by individual teachers wanting to conduct research in their own classrooms. He saw that the number of students in an individual teacher's classroom was never large enough to use available statistical procedures to determine the effectiveness of different instructional strategies or techniques tried or investigated by their teachers. Frymier, (Frymier, J., et al 1992 a) (Frymier, J., et al 1991) designed a single study that contained several different lines of research within the main study. The purpose of this design was to identify the factors that contributed to children being at risk of failing to complete a public school education and to gather a large enough data base to be able to study different lines of research within a main study. That single study was conducted within the same school year in 87 sites around the U.S. The study called "Multiple Replication" by Frymier (Frymier, J., et al., 1992b.) increased the number of students and dramatically produced unarguable results about the causes of students being at risk of failing in public schools.

*As of December 2006 Borenson and Associates reports having conducted live student demonstrations at more than 1500 **Making Algebra Child's Play** seminars held throughout the United States since January 1995.

The concept of multiple replication was tested again in a study (Barber, L.W., et al 1994.) that identified the causes of children being at risk in the USA versus being at risk in Russia. This study used a single design replicated 10 times in the US and 10 times in Russia.

The multi-site replications concept was pushed further in a study of the effects of a reading program called Reading Styles (RS) used by teachers in their individual classrooms (Barber, L.W., et al 1998.). No single classroom was large enough to prudently use the available statistics. The study taught individual teachers how to teach via RS and also taught them how to conduct their own research about the effects of their use of the RS method. The program managers in RS collected individual classroom data over time and as the numbers arrived at critical mass (large enough to use appropriate statistics), data analysis was conducted, and each year or at each new critical mass, the data are to be reanalyzed.

Individual program offices within the RS program began accumulating data on different student characteristics. Such characteristics as gifted students, learning disabled, handicapped, etc. were accumulated over time until they reached critical mass and then when the numbers were large enough to examine the teaching effect, proper statistical analysis was conducted.

The design called "Multi-site Replications and Multiple Student Characteristics Assessment" is at the heart of this study of HOE and is the design used herein to study both the teachers and students taught by this teaching program.

The Teacher Study

The purpose of this study is to ascertain the effects of learning the HOE method of algebra instruction in a **Making Algebra Child's Play** seminar on teachers who are taught this method.

Three components of this instructional effect will be examined.

The first component is designed to measure whether or not teachers are confident that they can teach algebraic concepts to at least 80% of the students in their lowest-achieving class using two different modes of instruction: the traditional approach to teaching these concepts, and HOE, and to determine if the seminar treatment has a significant effect on the confidence level. The pre-test inquired about the traditional approach and was administered prior to the beginning of the seminar. The post-test, inquired about the HOE approach, and was administered at the conclusion of the 6th lesson of the seminar.

The second component is designed to measure the self-reported ability level of teachers, on a rating from 1 to 10, with respect to their ability to teach algebraic concepts to their lowest achieving students, both prior to and after treatment, and to compare the results. The pre-test score was obtained prior to the beginning of the seminar. The teachers were not limited to any specific method of instruction in their pre-test response. They were to assess their current ability level using any method or knowledge at their disposal (at this point they did not know the HOE approach). The post-test score was obtained at the conclusion of the sixth lesson.

The third component was designed to measure the teachers' ability to solve a number of algebraic linear equations before the training and then immediately after the 6th lesson of the seminar

Statement Of The Problem

A problem is to identify whether or not teachers are confident that they can successfully teach algebraic concepts, and in particular the solution of algebraic linear equations, to at least 80% of the teacher's lowest achieving class, via two different modes of instruction: the traditional mode of instruction and HOE, and then to compare these responses to see if there is a significant difference. The response to the traditional mode was obtained on the pre-test, prior to the beginning of the seminar treatment. The HOE response was obtained at the conclusion of the 6th lesson of the **Making Algebra Child's Play** seminar.

A second problem is to identify the effect that learning the HOE method has on the teachers' self-concept about their ability to teach algebraic concepts, and in particular the solution of algebraic linear equations, to their lowest-achieving students, on a rating from 1 to 10. Self-concept was measured before and after the instructional treatment, consisting of the first six lessons of the seminar. For the pre-test measure the participants were not limited to any particular mode of instruction, as they were in the first problem above. The teachers were to rate their ability level using any method or knowledge at their disposal (at this point they did not know the HOE approach).

A third problem was to determine the change in the teachers' ability to solve a set of algebraic linear equations, before the instructional treatment and after the instructional treatment.

Limitations Of The Study

This study is not intended to learn all things about all teachers. The primary interest of the study is to learn the effect of the **Making Algebra Child's Play** seminar on the self-concept of a specific type of teacher. That is an elementary or middle school teacher in grades 3 to 8 who has come to a **Making Algebra Child's Play** seminar to learn how to teach algebraic concepts using HOE.

Currently for example we are not interested in studying the **Making Algebra Child's Play** seminar instructional effect on teachers who wave been taught HOE in previous years and are coming back to take a refresher course in HOE.

Currently for example we are not interested in studying the HOE instructional effect on teacher effectiveness while teaching algebra in comparison with other approaches those same teachers may have used in the past.

Currently for example we are not interested in studying the HOE instructional effect on teacher work satisfaction while teaching algebra.

Note: In the future we will be interested in studying the above examples of Teacher Characteristics as well as many more that will be considered.

The Student Study

The purpose of this study is to ascertain the effect upon students of being taught algebraic concepts via the methods of HOE by teachers trained in the use of the HOE method. The particular effect to be studied is student ability to solve algebraic linear equations before and after learning HOE.

This effect will be measured for specific groups and under specific conditions.

The first is to measure the effect on students in the regular education in public schools grades 3-8 who are taught HOE by teachers who were recently trained in the HOE method.

The second is to measure the effects on students in the special education program in public schools grades 4-12 who are taught HOE by teachers who were recently trained in the HOE method.

The third is to measure the effects on students in the gifted student program in public schools grades 2-6 who are taught HOE by teachers who were recently trained in the HOE method.

The fourth is to measure the effect on students in the regular education public school system, grades 3-8, who were taught to solve algebraic problems by teachers who are experienced in the HOE Method. For the purposes of this study, a teacher is considered "experienced" if he/she has taught the program for one year or longer.

Statement of the Problem

The problem is to determine if the HOE mode of instruction, taught by teachers trained in the method, leads to a significant gain in student achievement in solving algebraic linear equations.

Limitations of the Study

This study is not intended to learn all things about the effect the HOE method of instruction has upon students. The primary interest of the study is to learn the effect of the HOE method of instruction on student ability to solve algebraic linear equations.

Currently for example we are not interested in doing a comparative study of the effect of the HOE method of instruction vs. a traditional textbook method of instruction on student ability to solve algebraic linear equations.

Currently for example we are not interested in doing a comparative study to determine the effect of the HOE method of instruction on an end-of-year algebra test vs. students who have not been provided with the HOE treatment.

Currently for example we are not interested in doing a comparative study to determine the effect of the HOE method of instruction on standardized tests in mathematics vs. students who have not been provided with the HOE treatment.

Currently for example we are not interested in studying the effect of the HOE method of instruction longitudinally to see how these students perform in a regular algebra class several years later compared with students who did not have the HOE treatment.

Currently for example we are not interested in studying the effect of the HOE method of instruction on student self-esteem, interest in mathematics and general success in school.

Note: In the future we will be interested in each of the above examples as a line of research as well as many more lines of research as yet not considered.

Each of these lines of inquiry (with others to be added over time) will accumulate data until a critical mass is achieved, then data analysis will be conducted and the study data saved until the next critical mass is met.

Interim Results

This report is considered to be the first interim report. Given that the design of the study is one of "Multi-site Replication and Multiple Student Characteristics" we have already determined that data analysis will be conducted repeatedly as the number of subjects increase over time in each line of research. Slavin called this style of research a "multi-site replicated experiment" and described it thusly, "The defining characteristic of an MSRE is the pooling of data from several small experiments, each of which may be too small for adequate statistical power or generalizability, into one larger analysis. An effect size (Glass, McGow, and Smith, 1982) is computed for each experiment, and these are then pooled for the larger analysis. In fact, a multi-site replicated experiment may be seen as a form of meta-analysis and in a statistical sense this is true." (Slavin and Maddin, 1993)

The Teacher Study Results

First Problem: To obtain a measure of teacher confidence in teaching algebraic concepts to 80% or more of the students in their lowest achieving class using traditional instructional vs. HOE, and to compare the results.

We wished to identify whether or not teachers attending the **Making Algebra Child's Play seminar**, the large majority of whom are elementary and middle school teachers, are confident that they can successfully teach algebraic concepts to at least 80% of the teacher's lowest achieving class, via two different modes of instruction: the traditional mode of instruction and HOE, and then to compare these responses to see if there is a significant difference. The response to the traditional mode was obtained on the pre-test, prior to the beginning of the seminar treatment. The HOE response was obtained at the conclusion of the 6th lesson of the seminar, approximately 2 ½ hours into the full-day **Making Algebra Child's Play** seminar.

To accomplish this objective, the teachers were asked to respond anonymously to the following question on their onsite questionnaire (see Appendix), prior to the beginning of the seminar:

> *Please indicate "Yes" or "No" to the following question.*
>
> *"I am confident that using the traditional method of teaching algebra, I am able to teach 80% or more of the students in my lowest class how to understand and solve these two questions.*

$$2x + x + x + 2 = 2x + 10$$

and

$$2(x + 4) + x = x + 16$$

Following the conclusion of Lesson #6, they were asked to respond, also anonymously, to the following question on the same questionnaire:

> *Please indicate "YES" or "NO" to the following question:*
>
> *" I am confident that using the **Hands-On Equations** system of instruction, with each student having their own set of game pieces, that I would be able to teach 80% or more of the students in my lowest class how to understand and solve the two equations shown above."*

Both questions essentially asked the teacher, "Are you confident that you can teach these algebraic equations to 80% or more of the students in your lowest class using this specific method of instruction?" The teachers had to select either a "Yes" or a "No" response.

For this line of research we found 751 teachers who filled out both the pre-test and the post-test. All but 46 of these respondents were elementary and middle school teachers. In order to quantify the data we assigned a 1 to each "yes" response on the pre and post-test and assigned a 0 to each "no" response. The comparison was between the mean of the yeses on the pre-test and the mean of the yeses on the post-test. The pre-test mean was .162. The post-test mean was .984. We then calculated a t value (we used the t for paired observations) between the two means: the t value was 57.81. The post-test mean was over 6 times larger than the pre-test mean.

Teacher Study Data Results: Problem #1

	X1	X2	D	D2
Sum	122	740	618	622
Mean	.162	.984		
N =	751			
t =	57.81			

The statistic used above was the difference between the Means for Paired Observation and Equated Groups ("t-test for paired observation," Edwards 1963) (Barber et. al 1988). The formula used came from the Edwards book (p. 281) and was applied to test the difference between the group mean on the pre-training self-report and the group mean on the post-training self-report from the same teachers. For this first analysis we simply analyzed the data on all teachers who gave both a pre and post response.

<u>**Conclusion**</u>**:** *We note that only 16% of teachers coming to the* ***Making Algebra Child's Play*** *seminar expressed confidence that they would be able to teach 80% or more of the students in their lowest classes the solution to equations such as* **$2x + x + x + 2 = 2x + 10$ and $2(x + 4) + x = x + 16$** *using the <u>traditional</u> teaching methods.* In light of the significant relationship which research shows to exist between teacher efficacy, i.e. teacher belief, and student achievement (see page one of this study), this result is important. If this result turns out to be representative of teachers nationwide, it would suggest that the use of the traditional methods of instruction is not likely to accomplish the goal of successfully teaching the above concepts to 80% or more of the students in our lowest achieving classes. On the other hand, *by the end of the 6th lesson of the seminar, 98% of the participants at this seminar, the majority of who were elementary and middle school teachers, expressed confidence that, using the Hands-On Equations system of instruction, with each child having their own set of manipulatives, they would be able to teach these concepts to 80% or more of the students in their lowest class.*

Second Problem: To ascertain the level of teacher self-concept of their ability to teach algebraic concepts to their lowest achieving students, prior to and after treatment, and to compare the results.

We wished to identify the effect of the **Making Algebra Child's Play** seminar on the teacher's self-concept about their ability to teach algebraic concepts, and in particular the solution of algebraic linear equations, to their lowest-achieving students (and not merely to 80% of their lowest class, as was the case with the first question). In addition, for the pre-test, the teachers were not limited to the use of any specific mode of instruction (as they had been with the first question). Rather, the teachers could use any method at their disposal (they were not yet familiar with HOE). This self-concept of ability level was measured by a self-report contained within the anonymous onsite questionnaire that was used for both studies. The pre-test was administered at the beginning of the seminar; the post-test was administered at the conclusion of Lesson #6, approximately 2 1/2 hours into the seminar.

The teachers were presented with the following two algebraic linear equations.

$$\mathbf{2}\boldsymbol{x} + \boldsymbol{x} + \boldsymbol{x} + \mathbf{2} = \mathbf{2}\boldsymbol{x} + \mathbf{10}$$
and
$$\mathbf{2(}\boldsymbol{x} + \mathbf{4)} + \boldsymbol{x} = \boldsymbol{x} + \mathbf{16}$$

The pre-training instruction, provided on the onsite questionnaire, was:

"On a rating from 1 – 10, with 1 the lowest and 10 the highest, please rate your **current** ability level to successfully teach the above concepts to your lowest achieving students:"

1-- 2-- 3-- 4-- 5-- 6-- 7-- 8-- 9-- 10--

Their post-training instruction, provided on the onsite questionnaire, was:

"You have now completed the first six lessons of Hands-On Equations. On a rating from 1 – 10, with 1 the lowest and 10 the highest, please rate your ability level **now**, to teach the above concepts to your lowest achieving students:"

1-- 2-- 3-- 4-- 5-- 6-- 7-- 8-- 9-- 10--

For this interim report we have the analysis of two sets of teacher self-report data. The first was conducted when we had obtained data on the first 129 teachers who had submitted the questionnaires; the second was conducted when we had obtained similar data on a total of 814 teachers, including the original group.

The statistic we used was "t-test for paired observation." Note: since we were using a Paired Observation statistic all pairs had to have complete data. Due to this rule we had to eliminate 10 teachers from the first group of 129 reducing the number to 119. We had to eliminate 67 teachers in the second group of 813 who did not provide a number on the pretest or post-test. We also had to eliminate 7 who placed a double entry on the pre-test or post-test.

Teacher Study Data Results: Problem #2

First run of teacher data, N of 119 teachers

	X1	X2	D	D2
Sum	618	1026	408	2084
Mean	5.19	8.62		
N =	119			
t=	15.5, sig. at .01 level or beyond			

Second run of teacher data, N of 739 teachers

	X1	X2	D	D2
Sum	3835	6522	2687	13689
Mean	5.19	8.83		
N =	739			
t =	42.90, sig. at .01 level or beyond			

We note the consistency of these results, even though the N jumped from 119 to 746. The pre-test means of 5.19 did not budge from the smaller sample to the much larger one. The post-test mean of 8.62 obtained with the smaller sample increased to 8.83 with the larger sample. Looking at the larger sample, prior to the seminar, the teachers rated their ability level to teach algebraic concepts to their lowest achieving students as 5.19 out of a possible 10. After the 6^{th} lesson of the seminar, they rated their ability level to teach algebraic concepts to their lowest-achieving students at 8.83 out of a possible 10. The gain was statistically significant.

<u>Conclusion</u>: Teachers attending the **Making Algebra Child's Play** seminar reported a significant increase in their self-perceived ability to teach algebraic concepts to their lowest achieving students, as measured by the post-test provided after the 6^{th} lesson. Numerically, on a rating from 1 to 10, with 10 the highest, the pre-test average was 5.19, whereas the post-test average was 8.83. Since a significant relationship exists between teacher efficacy and student achievement (Kearns, 1995), this increase in teacher-reported ability level is expected to have a positive, significant effect on student achievement if the teachers were to use the Hands-On Equations program in their classrooms.

The third problem in the teacher study was to determine the change in the teacher's ability to solve a set of algebraic linear equations, before and after the treatment. At this date we do not have sufficient data to analyze to examine this problem.

<u>General Conclusion Of Teacher Study</u>

. On the pre-test, only 16% of teachers coming to a **Making Algebra Child's Play** seminar expressed confidence of being able to successfully teach 80% or more of the students in their lowest-achieving class algebraic concepts such as those found in $\mathbf{2x + x + x + 2 = 2x + 10}$ **and** $\mathbf{2(x + 4) + x = x + 16}$ using the traditional notation.

. On the post-test, at the conclusion of Lesson #6, 98% of the seminar participants, the large majority of who are elementary and middle school teachers, expressed confidence that using HOE, with each student having their own set of manipulatives, they would be able to successfully teach these concepts to 80% or more of the students in their lowest achieving class.

. The teacher self-reported rating of their ability to teach the above algebraic concepts to their lowest achieving students increased from 5.19 to 8.83 after being exposed to the first six lessons of the **Making Algebra Child's Play** seminar. This change was found to be statistically significant.

. Since a significant relationship exists between teacher efficacy and student achievement, it would be expected that the positive impact of the **Making Algebra Child's Play** seminar on teacher self-concept of teaching ability would translate into learning success for students in whose classes these teachers use HOE to teach algebraic concepts.

<u>The Student Study</u>

The Problem: The problem is to determine if the HOE mode of instruction, taught by teachers trained in the method, leads to a significant gain in student achievement in solving algebraic linear equations.

Six questions of varying difficulty were selected to measure the effect of the treatment. For <u>each</u> of these questions, four additional questions of the same type were formulated. We now had available 5 sets of six questions each with which to test the treatment effect. A table of random numbers was then used to select which version of question #1 through #6 would appear on the pre-test, and which version would appear on the post-test. <u>Note</u>: This procedure was followed to prevent unintended bias in the selection of the version of the question that would appear on the pre-test and those that would appear on the post-test. The versions of the questions chosen by this procedure are listed below. The students were provided with 15 minutes each within which to do the pre and the post-test. For the post-test, the students were allowed to use their manipulatives.

Pre-Test Questions	**Post-Test Questions**
1. $2x = 8$	**1.** $2x = 10$
2. $x + 3 = 7$	**2.** $x + 3 = 8$
3. $2x + 2 = 8$	**3.** $2x + 2 = 10$
4. $3x = x + 6$	**4.** $3x = x + 4$
5. $4x + 3 = 3x + 6$	**5.** $4x + 3 = 3x + 9$
6. $2(2x + 1) = 2x + 6$	**6.** $2(2x + 1) = 2x + 8$

The Student Study Results

As of this writing we have received student data from only one experienced teacher that is pertinent to the question of a regular student's ability to solve algebraic linear equations before and after learning HOE. 18 regular education 5th grade students took part in this study. The teacher had 4 gifted and 2 LD students in her class whose data were not included in this line of research.

We also have data for one class of students in a special-ed class grades 9 and 11 combined. There are 16 students in grade 9 and 7 students in grade 11. N=23.

These two sets of data represent two lines of research and we will analyze the data for each class separately. Note: We are aware that the numbers are too small to generalize beyond each classroom.

The Williford Study

This study contained 18 students in regular education grade 5. The pre-test was administered on Oct. 23, 2006; the post-test was administered on Nov. 6, 2006 after instruction in the HOE method.

N = 18 students

	X1	X2	D	D2
Sum	42	105	63	237
Mean	2.33	5.83		
N=	18			
t=	15.10 sig. @ .01 beyond			

The mean score for these 5th grade students increased from 39% on the pre-test to 97% on the post-test. From this single study one can conclude that regular education students can be successfully taught algebraic concepts via Hands-On Equations to the extent that a significant gain is shown from a pre-test to a post-test. We notice that the students of this class achieved almost perfect mastery of the concepts as measured by the post test.

The Ankney Study

This study has an N of 23. There were 16 students in grade 9 and 7 students in grade 11 in the Ankney classroom. All students are listed as L.D. The pre-test was administered on Nov. 6, 2006 and the post-test was administered on Nov. 22, 2006, after instruction in the HOE method.

N = 23 students

	X1	**X2**	**D**	**D2**
Sum	65	109	44	148
Mean	2.83	4.74		
N=	23			
t=	5.38 sig. @ .01			

The mean score for these high school students classified as LD, increased from 47% on the pre-test to 79% on the post-test. From this single study one can conclude that high school L.D. students can be successfully taught algebraic concepts via Hands-On Equations to the extent that a significant gain is shown from a pre-test to a post-test. Nonetheless, the gain shown by these high school LD students was not as strong as that shown by the 5th graders, nor did they achieve the same level of mastery as the younger students.

The Barclay Study

In a previous study (Barclay 1992) Barclay examined the effect of teaching with the HOE method. Barclay administered a 10 item pre-test. She then assisted 5 classroom teachers (all had been previously trained in HOE) in teaching their students the HOE method of algebra. She then conducted post-test and retention assessment at 3 weeks and 6 weeks after post-testing on 6 classes of children, totaling 123 students age 11 -13.

Barclay concluded on pages 35 – 36: "Given the pre-test data and the post-test, three-week retention test, and the six-week retention test data, it can be concluded that each student learned. 100% of the students demonstrated at least 80% mastery on at least two out of the three tests that followed the instruction.

"The increase in scores from pre-test to post-test is notable but even more worthy of note is the relative stability and maintenance of mastery level scores. The strength of the study is not simply that the students learned with the manipulatives, but that they were able to retain the concept without further instruction or, given only the prompt of the manipulative materials during the test administration."

Conclusion of Student Study One can conclude only that each HOE classroom where we have accumulated data under research conditions shows a statistically significant gain from pre-test to post-test and where gathered show retention of what was learned. Our purpose is to continue to accumulate data on the various lines of research we are engaged in, and to begin new lines of research, until each reaches a critical mass and then analyze the data to further determine the effect of HOE.

Overall Conclusion of Present Study Only 16% of 751 teachers attending the **Making Algebra Child's Play** seminar expressed confidence that using the traditional teaching methods they would be able to teach algebraic concepts such as $2x + x + x + 2 = 2x + 10$ and $2(x + 4) + x = x + 16$ to 80% or more of the students in their lowest achieving class. After participating in the first six lessons of the seminar, approximately 2½ hours into the seminar, 98% of the teachers attending the seminar, the large majority of whom were elementary and middle school teachers, expressed confidence in being able to teach these concepts to 80% or more of the students in their lowest-achieving class if they were to use Hands-On Equations, with each student having their own set of manipulatives. Additionally, after the completion of the sixth lesson, the teachers reported a significant gain in their self-perceived ability level to teach the above concepts to their lowest achieving students. The student test results showed that the students who were taught algebraic linear equations using HOE, by teachers trained in the HOE method, achieved significant gains in solving algebraic linear equations as measured by the pre and post-test**.**

Larry W. Barber served as Director of Research for Phi Delta Kappa from 1983 to 1999.
Henry Borenson developed the Hands-On Equations Program. He is President of Borenson and Associates, Inc.

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Appendix A

Hands-On Equations Workshop
Onsite Questionnaire

October 27, 2006 **Knoxville, TN** **Dr. Joyce Hardaway**

Dear Seminar Participant:

1. Please check the grade level you teach: elementary ____ middle school ____ high school____

2. Please indicate "YES" or "NO" to the following question:

   "I am confident that using the traditional methods of teaching algebra I am able to teach 80% or more of the students in my lowest class how to understand and solve these two equations:

$$2x + x + x + 2 = 2x + 10$$
and
$$2(x + 4) + x = x + 16$$

   YES _____ NO _____

3. Do you have a major, minor or certification in mathematics? Yes __ No __

4. If you would like to solve these two equations, you may do so:

   a) **$2x + x + x + 2 = 2x + 10$, x =** _____ **Check:** ______

   b) **$2(x + 4) + x = x + 16$. x =** _____ **Check:** ______

5. On a rating from 1- 10, with 1 the lowest and 10 the highest, please rate your **current** ability level to successfully teach the above concepts to your lowest-achieving students:
   1__ 2__ 3__ 4__ 5__ 6__ 7__ 8__ 9__ 10__

PLEASE ANSWER THE FOLLOWING QUESTIONS
AT THE END OF LESSON #6 IN THE MORNING SESSION

6. Please indicate "YES" or "NO" to the following question:

   "I am confident that using the **Hands-On Equations** system of instruction, with each student having their own set of game pieces, that I would be able to teach 80% or more of the students in my lowest class how to understand and solve the two equations shown above:

   YES _____ NO _____

7. Please use your Hands-On Equations kit to provide your answers to these two questions:

   1. $2x + x + x + 2 = 2x + 16$ **1. x =_____ Check: ___**

   2. $2(x + 4) + x = x + 20$ **2. x =_____ Check: ____**

8. You have now completed the first six lessons of Hands-On Equations. On a rating from 1- 10, with 1 the lowest and 10 the highest, please rate your ability level **now** to teach the above concepts to your lowest-achieving students:
   1__ 2__ 3__ 4__ 5__ 6__ 7__ 8__ 9__ 10__