Suggestions for Teaching Verbal Problems
Using Hands-On Equations®

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Careful Reading of the Problem

The solution of verbal problems requires careful reading of the problems. There are no short cuts available, such as "clue words" or "key words." The student must read and interpret the problem. Usually, this will require several readings of the problem as the student tackles one aspect of the problem and then the next aspect. Consider the example:

> *One hen laid twice as many eggs as a younger hen. This gave the farmer just enough eggs to sell 9 and keep 9. How many eggs did each hen lay?*

The student needs to make sense of this problem. He/she has to process the information provided. The student has to realize that the total number of eggs laid by both hens is 18. (The problem does assume that each egg laid was either kept or sold, and that none were rejected for any reason. The problem also assumes the farmer had no eggs to start with. A student who makes these observations should be praised for his/her careful reading of the problem.)

Representing the Unknown Fully and in Writing

An essential step to the solution of verbal problems is to specify what the blue pawn is to represent as clearly as possible. The teacher should encourage an accurate written representation. Abbreviations at this stage can become a big stumbling block.

Consider the example:

> *Mary went to the store and spent $12. She bought 3 equally priced notebooks. How much did each one cost?*

The student may think he/she wants the pawn to represent the notebook, and so may write "♟: notebook". It should be pointed out to the student that the notebook has many attributes, such as length, width, weight, age, etc. Hence, we ask the student, "Exactly what aspect of the notebook is of interest to us?" The student will see that we are interested in the price of the notebook. Hence, the student should write "♟: price of a notebook", or even better, "♟: price of a notebook in dollars".

(Nevertheless, a student who uses the blue pawn to represent "the notebook," and at the end proceeds to indicate that each notebook costs $3, should be given credit for his/her solution even though the representation was not written in the most specific manner possible. This recommendation applies especially in the early stages of the student's work with verbal problems.)

Representing Even Simple Problems Using the Pawns and Cubes

The problem on the previous page is one that many students will be able to solve mentally. Nonetheless, the student should be challenged: "If we wish to represent this problem with the pawns and cubes, how would we do so? What would you like the blue pawn to represent?" This exercise forces the student to break down and verbalize their thinking. It also forces them to think about representing abstract concepts concretely and it is an exercise that is highly recommended.

When a Student Has Difficulty Understanding the Verbal Problem

A student who does not know how to proceed, should attempt to guess and test an answer to the problem to give him/her "a feeling" for the problem. Consider the following example.

> *When a number is increased by 2, and the sum obtained is doubled, 16 is obtained. What is the number?*

We ask the student to select any number. Say the student picks the number 4. We then ask, "What is 4 increased by 2?" (6) "When that is doubled, what do you get? (12) "Does 4 work? We were supposed to get 16 as the result." (4 does not work)

We ask the student to try another number. He/she selects 5. We ask, "What is 5 increased by 2?" (7) What do you get when you double that?" (14). "Does 5 work?" (No)

In this manner the student begins to understand what the problem is asking, and can set up the problem as shown below.

Solution:

Let the number be: ▲

Then, the number increased by 2 would be: ▲ [2]

▲ [2] ▲ [2] ⊥ [10] [6]

▲ = 6

Answer: The number is 6. Check: 16 = 16.

Representing Several Unknowns in One Problem

> *Christy is 3 years older than Jamie. The sum of their ages is 33. How old is each?*

From the earlier discussion, we know that it is not sufficient for the student to let the blue pawn represent Jamie. Jamie has many characteristics such as weight and height. Hence, the correct representation is that the blue pawn should represent Jamie's age. This is written, "▲: Jamie's age". We can be even more specific and specify that we want the pawn to represent Jamie's age in years, but this is understood and so need not be specified.

In this example, it is natural to let the blue pawn represent Jamie's age since we know less about Jamie. Christy's age is given in reference to Jamie. Hence, once we know that Jamie's age is represented by a blue pawn, we can then represent Christy's age with a blue pawn and a red 3 cube, "▲ [3] : Christy's age". As in this example, a student should write down a representation for each unknown in a problem with several unknowns. If the student has difficulty with writing or is lazy about doing so, the teacher can write down these representations on the blackboard. With these representations in front of them, the students are ready to attempt to set up the problems. After figuring out the value of the blue pawn, they can also go back to the representations written on the blackboard or in their notebooks to figure out the value of each of the unknowns.

Representing the Unknowns Before Attempting to Set Up the Problem

The student should form the habit of representing the unknowns on paper prior to setting up the problem on the laminated scale with the game pieces or setting it up pictorially.

Consider the following example*:

> *Find three consecutive numbers such that triple the last number gives the same result as twice the sum of the first two numbers. (PICTORIAL)*

Three consecutive numbers can be represented, respectively, by a blue pawn, a blue pawn and 1 red cube, a blue pawn and a red 2 cube. If the student started by placing all these three elements onto the left side of the balance scale, he/she may think they belong there. On the other hand, if the student represents the elements in writing on paper, as follows:

Let the 1st number be: ▲
Let the 2nd number be: ▲ [1]
Let the 3rd number be: ▲ [2]

and only then begins to do the setup, the student will see that only the last number, set up three times, appears on the left side of the setup. Whereas the right side would contain the sum of the first two numbers, set up twice. Hence, the pictorial setup for this verbal problem would look as follows:

▲ [2]   ▲ [2]   ▲ [2] ⊥ ▲ ▲ [1]   ▲ ▲ [1]

* Suggestions for working with consecutive numbers are given later in this section.
Note: In this book, a pictorial solution is recommended when there are not sufficient game pieces in one student packet to represent the problem. Some pictorial problems can be solved with the game pieces if two student packets are used.

Submitting the Answer to a Verbal Problem

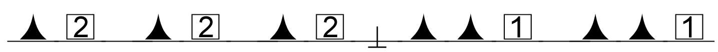
Solving the above setup mentally, we see that the value of the blue pawn is 4. The student will write this down as "▲=4" or $x = 4$. At this point, the answer is not complete, since the student has not answered the question which was posed, namely, to find the three consecutive numbers meeting the conditions of the problem. We can easily do this, however. Since the blue pawn represents the first of these three consecutive numbers (see above representation), we conclude that the three numbers are 4, 5 and 6. The student should be asked to write out the final answer to the problem long hand: "The three consecutive numbers are 4, 5 and 6."

Conducting the Check to a Verbal Problem

As is the case throughout Hands-On Equations, the student is encouraged to do the check. The check in the verbal problem will usually be done by students in the original physical or pictorial setup. If the student has removed pieces to solve the setup, he/she will need to reset it. Let us look at the above example one more time.

Find three consecutive numbers such that triple the last number gives the same result as twice the sum of the first two numbers. (PICTORIAL)

Let the 1st number be: ▲
Let the 2nd number be: ▲ [1]
Let the 3rd number be: ▲ [2]



▲= 4

Answer: The three numbers are 4, 5 and 6.

Looking at the above setup, with the pawn having a value of 4, we see that both sides have a value of 18. Hence, check: 18 = 18.

Stronger students will be able (and should be encouraged) to do the check in the original verbal statement of the problem: triple the last number (6) gives 18. The sum of the first two numbers is 9. Twice that sum is 18. Hence both sides have the same value, 18 = 18.

Verbal Problems Involving Fractional Relations

The teacher has an opportunity to reinforce basic fractional relations in working with problems such as the following:

Half of a number, increased by 4, is 18. Find the number.

The students should be given an opportunity to work on this problem without any suggestions from the teacher. It is likely that in a class of students, one of them will come up with the answer by trial and error, or will suggest that we use two game pieces to represent the problem. If no suggestions are forthcoming from the class, the teacher can ask a student to select a number at random. Say a student selects the number 6. The teacher can now ask, "The problem says 'half the number.' What is half of 6?" (3) "What is 3 increased by 4?" (7) "Did 6 work?" (No, we are supposed to get 18.)

Let's say the student next tries the number 10. The teacher can ask, "What is half of 10?" (5) "What is 5 increased by 4?" (9) "Did 10 work?" (No)

After a few such examples the teacher can ask the students, how they would like to represent the number? A student may suggest that we can represent the number by <u>two</u> blue pawns. Half of the number would then be represented by one blue pawn.

<u>Solution:</u>

Since we will need half of the number, it is best to represent the original number by <u>two</u> blue pawns

Let the number be: ▲▲

Half of the number is: ▲

Half of the number increased by 4 is: ▲ [4]

Hence, the setup for the above problem is:

▲ [4] ⊥ [10] [8]

▲=14

Once we obtain the value of 14 for the blue pawn, we look at our representation which tells us that the original number was represented by two blue pawns. Hence, the original number must be 28.

<u>Answer:</u> The number is 28. Check: 18 = 18.

These verbal problems with fractional relations will reinforce the basic fractional relations, such as 1/2, 1/3, 2/3, 1/5, 2/5 3/5 and 4/5. For example, a problem dealing with thirds would be approached by letting the unknown number be represented by 3 blue pawns. Each blue pawn would then represent one-third of the number. The student simply needs to remember that once the value of the blue pawn is obtained, that value needs to be multiplied by 3 to get the original number. Students who have worked with Level I of Hands-On Equations should be able to experience success with these verbal problems.

<u>Verbal Problems Involving Consecutive Numbers</u>

In introducing this topic, the teacher should ask for an example of a set of three consecutive numbers. Say the students have given the following set of three consecutive numbers: 4, 5 and 6.

In order to draw out from the student the abstract relationship between consecutive numbers, the teacher can ask, "How much do we need to add to one number of this set to get the next consecutive number?" The students should see that these numbers differ by 1. Hence, if the first number is represented by a blue pawn, the next consecutive number will be represented by a blue pawn and a 1 red cube, and the consecutive number after that by a blue pawn and a 2 red cube:

Let the 1st number be: ▲

Let the 2nd number be: ▲ [1]

Let the 3rd number be: ▲ [2]

The above numbers, since they differ by 1, can serve as a representation for any three consecutive numbers.

The teacher can now ask for a set of three consecutive even numbers. Say the students respond with: 8, 10 and 12. The teacher can again ask the students to state the relationship between any two consecutive numbers of this set. The students should see that they differ by 2. Hence a representation for three consecutive even numbers is:

Let the 1st number be: ▲
Let the 2nd number be: ▲ [2]
Let the 3rd number be: ▲ [4]

The teacher can now ask for a set of three consecutive odd numbers. Say the students respond with: 5, 7 and 9. The teacher can ask the students to state the relationship between any two consecutive numbers of this set. The students should see that these also differ by 2! Hence a representation for three consecutive odd numbers is:

Let the 1st number be: ▲
Let the 2nd number be: ▲ [2]
Let the 3rd number be: ▲ [4]

We see from the above that consecutive even numbers, and consecutive odd numbers, since they differ by 2, can both be represented in the same manner! Once the students can make these representations they are ready to work with even "sophisticated" consecutive number problems, such as the one presented earlier.

> *Find three consecutive numbers such that triple the last number gives the same result as twice the sum of the first two numbers. (PICTORIAL)*

By introducing consecutive numbers in the above manner, the teacher will find that even his/her 4th and 5th grade students can be successful with these concepts, provided the students have completed Level I of Hands-On Equations. Whereas it is true that students in a regular 9th grade algebra class may have difficulty with this problem (and others presented in this book), the power to simplify the concepts and make them concrete that is provided by Hands-On Equations makes these concepts accessible years earlier than would otherwise be the case. Success with these concepts will empower the young student and raise his/her mathematical aspirations.

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