Research-Informed Answers for Mathematics Education Leaders

LEADERSHIP IN MATHEMATICS EDUCATION
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Improving Student Achievement in Mathematics by Using Manipulatives with Classroom Instruction

Manipulative materials help students make sense of abstract ideas, provide students ways to test and verify ideas, are useful tools for solving problems, and make mathematics learning more engaging and interesting by lifting mathematics off textbook and workbook pages.

Burns, 2007

Our Position

It is the position of the National Council of Supervisors of Mathematics (NCSM) that in order to develop every student's mathematical proficiency, leaders and teachers must systematically integrate the use of concrete and virtual manipulatives into classroom instruction at all grade levels. This position can be accomplished when leaders and teachers:

- Understand that manipulatives are not toys but are powerful learning tools which build conceptual understanding of mathematics;
- Use research to guide instructional use of manipulatives;
- Provide sustained professional learning opportunities in the use of manipulatives; and
- Recognize that learners—both adults and students—progress through varying levels of proficiency as they use manipulatives before they can realize their full impact.

The *Common Core State Standards* emphasize that concrete models are an essential tool for learning mathematics across all grade levels, K–12. This assertion is articulated most clearly in the Standard for Mathematical Practice 5, "Use Appropriate Tools Strategically," where students choose from concrete models (including manipulatives) and technology. Beyond this, the standards regularly suggest using models in initial steps of learning mathematics before students move to other representations. Therefore, students should have a variety of manipulatives and tools available to them at all times.

Manipulatives used in classroom instruction are physical objects handled by individual students and small groups. Virtual manipulatives are important tools for teacher modeling and demonstration and, additionally, provide students access to manipulatives both inside and outside of the school day via computers. However, virtual manipulatives do not replace the power of physical objects in the hands of learners.

Research That Supports Our Position

John van de Walle and his colleagues (2012) define a mathematical tool as, "any object, picture, or drawing that represents a concept or onto which the relationship for that concept can be imposed. Manipulatives are physical objects that students and teachers can use to illustrate and discover mathematical concepts, whether made specifically for mathematics (e.g., connecting cubes) or for other purposes (e.g., buttons)" (p. 24). Moyer et al. (2000) define virtual manipulatives as "digital objects that resemble physical objects and can be manipulated with a mouse of a computer" (p. 372). For example, virtual versions of Cuisenaire Rods and Tangrams are readily available online for instructional purposes.

In the opening quote, Marilyn Burns (2007) provides four reasons manipulative materials are fundamental to mathematics instruction. These ideas appear repeatedly

The NCSM Improving Student Achievement Series is a set of position papers designed to provide research-based practices for school and district mathematics education leaders.

in research and thoughtful commentary on the teaching of mathematics. The National Research Council's *Adding It Up* (2001) concludes its review of research on the role of manipulatives with the following statement:

> "The evidence indicates, in short, that manipulatives can provide valuable support for student learning when teachers interact over time with the students to help them build links between the object, the symbol, and the mathematical idea they represent" (p. 354).

Numerous studies have examined the effectiveness of specific manipulatives to teach specific topics. For example, the Milken Family Foundation analysis of NAEP data suggests that the use of hands-on materials is highly effective. The findings note that "when students are exposed to hands-on learning on a weekly rather than a monthly basis, they prove to be 72% of a grade level ahead in mathematics" (Wenglinsky, 2000, p. 27). Additionally, Sowell (1989) conducted a meta-analysis of studies focused on teaching with manipulatives and found them to have a positive impact on mathematics learning. Cramer et al. (2002) compared the performance of 1,600 fourth and fifth grade students studying fractions using both manipulative-based curricula and non-manipulative based curricula. They found that students in the manipulative-based program had higher mean scores at the end of the unit as well as higher retention scores.

Manipulatives are also considered an important element of teacher preparation. For example, the Conference Board of Mathematical Sciences' 2012 report, the *Mathematical Education of Teachers II,* includes numerous references to the use of manipulatives in classroom instruction and the importance of teacher preparation for this use. The authors continue by pointing out that teachers must work to help students see the connections between the manipulatives or other tools and the mathematical concept being taught. A number of studies cited in Van de Walle et al. (2012) suggest that manipulative instruction which follows a pattern of "do as I do" is one of the most widespread misuses of manipulatives. Stein and Bovalino (2001), for example, suggest three key features of successful manipulative lessons that avoid this pitfall. They conclude that 1) teachers have extensive training in the use of manipulatives; 2) teachers prepare by using manipulatives to complete the same instructional activities they would ask of their students; and 3) teachers prepare the classroom for activities by organizing students in groups, preparing materials, and thinking through the logistics of the lesson.

Similar findings on the importance of effective instructional strategies when teaching with manipulatives appear in the 2009 Institute for Education Sciences report on response to intervention in mathematics (Gersten et al., 2009). The report states that "research shows that the systematic use of visual representations and manipulatives may lead to statistically significant or substantively important positive gains in math achievement" (pp. 30–31). The report goes on to discuss the importance of transitioning from concrete objects to visual representations and then to abstract notation. It provides a comprehensive summary of the evidence supporting the use of manipulatives, including evidence supporting the Concrete—Representational—Abstract (CRA) method of instruction. This method, grounded in Bruner's (1966) constructivist discussion of enactive/iconic/symbolic progression in learning, provides a basis for an effective framework for teaching with manipulatives. Under this framework, teachers begin with concrete manipulative experiences, transition students to using visual representations (drawings), and finally transition to using abstract mathematical notation.

Hattie (2012) states "when teachers see learning occurring or not occurring, they intervene in calculated and meaningful ways. In particular, they provide students with multiple opportunities and alternatives for developing learning strategies based on the surface and deep levels of learning some content or domain matter, leading to students building conceptual understanding of this learning, which the students and teachers then use in future learning" (p. 15). Hattie later cites research on the power of balance in the classroom: "There is a balance between teachers talking, listening, and doing; there is a similar balance between students talking, listening, and doing" (p. 76). Manipulatives provide a foundation around which teachers and students can talk, listen, and do. Other research from Hattie (2009) concludes that, more often than not, when students do not learn, they do not need "more;" rather, they need "different" (p. 83). Again, to ensure that every student learns mathematics, a wide range of different strategies are needed for teaching and both physical and virtual manipulatives are a critical part of this toolkit.

Witzel et al. (2003) describe an example of successful implementation of the CRA approach in teaching algebra to middle grades students. The Association of Middle Level Education's research summary, Manipulatives in Middle Grades Mathematics (Goldsby, 2009), provides further information about this and other studies.

How NCSM Members Can Implement Our Position

As leaders, NCSM members must work to ensure that research-based recommendations are implemented in their schools, districts, states, and provinces. NCSM members must act to create and sustain the conditions and structures that will enable every mathematics teacher to use manipulatives successfully. Moreover, NCSM members must act to alert teachers, coaches, and administrators that it is time to move away from incidental to systematic approaches to manipulative-based instruction. NCSM members must act to move communities away from the "yes, but … I learned math without manipulatives" or, "they're playing with toys instead of learning mathematics" towards the power of multi-modal, conceptually based, hands-on instruction.

More specifically, NCSM members must:

- Ensure that curriculum documents K–12 support the use of manipulatives by their inclusion as an instructional tool on par with textbooks, technological tools, or other resources;
- Ensure access to manipulatives for every teacher and every student;
- Ensure ongoing professional development around the use of manipulatives;
- Ensure that teachers work collaboratively on grade level or subject level teams to provide equity among all student opportunities in using manipulatives;
- Ensure that the use of manipulatives is not viewed as optional by teachers, while recognizing that the nature and frequency of use will vary from course to course;
- Ensure the support of manipulatives to scaffold learning and in the problem-solving process;
- Ensure that teachers use manipulatives within the Concrete—Representational—Abstract Learning Cycle;
- Ensure that parents are educated about the place of manipulatives in the mathematics classroom;
- Ensure that manipulative-based activities are used for formative assessment in classrooms;
- Ensure that student background knowledge is considered in the variety of student choices; and
- Ensure that students have manipulatives available to help provide evidence in visualizing their thinking.

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National Council of Supervisors of Mathematics

<u>Mission Statement</u>

The National Council of Supervisors of Mathematics (NCSM) is a mathematics leadership organization for educational leaders that provides professional learning opportunities necessary to support and sustain improved student achievement.

<u>Vision Statement</u>

NCSM envisions a professional and diverse learning community of educational leaders that ensures every student in every classroom has access to effective mathematics teachers, relevant curricula, culturally responsive pedagogy, and current technology.

To achieve our NCSM vision, we will:

N: Network and collaborate with stakeholders in education, business, and government communities to ensure the growth and development of mathematics education leaders

C: Communicate to mathematics leaders current and relevant research; and provide up-to-date information on issues, trends, programs, policies, best practices and technology in mathematics education

S: Support and sustain improved student achievement through the development of leadership skills and relationships among current and future mathematics leaders

M: Motivate mathematics leaders to maintain a life-long commitment to provide equity and access for all learners

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