

The Effect of  
Hands-On Equations<sup>®</sup>  
on the Learning of Algebra by  
6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> Grade  
Inner City Students

by Henry Borenson and Larry W. Barber

Hands-On Equations Interim Report: December 1, 2008

*A Study to Determine if there is a Difference in Performance Among Students in Grades 6 – 8 by  
Grade Level, Special Group Classification, Gender or Race*

# TABLE OF CONTENTS

Abstract .....	1
General Introduction .....	2
Hands-On Equations .....	2
Meta-Analysis Approach .....	2
Definition of Terms .....	3
Description of Tests Used in the Study.....	4
Prior Studies.....	4
Rationale of the Study .....	5
Purpose of the Study .....	5
Teachers of the Study .....	6
Students of the Study .....	6
Classes of the Study .....	6
Classroom Instruction Used in the Study .....	7
Testing Procedure .....	7
Study Hypotheses .....	8
Results .....	8
Conclusions .....	11
Summary: .....	16
The Authors of the Study .....	18
References .....	19
Appendix 1: An Example of Hands-On Equations .....	20
Appendix 2: HOE Teaching Objectives .....	21
Appendix 3: Example of Pictorial Notation .....	22
Appendix 4: Test Questions Used in the Study .....	23
Appendix 5: Class not Included in the Study.....	24
Appendix 6: Test Items Analysis Study 158MA 6 <sup>th</sup> Grade Inner City.....	25
Appendix 7: Test Items Analysis Study 160MA 7 <sup>th</sup> Grade Inner City.....	26
Appendix 8: Test Items Analysis Study 159MA 8 <sup>th</sup> Grade Inner City.....	27
Appendix 9: Test Items Analysis Study 150C 7 <sup>th</sup> Grade Inner City GT.....	28
Appendix 10: Pre-test vs. Post-Test: Percentage of Correct Responses/Item ...	29

## ABSTRACT

A middle school in a large school district in the northeast United States agreed to participate in a research study to determine the effectiveness of the Hands-On Equations<sup>®</sup> program on student achievement in algebra by grade level (6<sup>th</sup>, 7<sup>th</sup>, or 8<sup>th</sup>), by group classification (LD, ELL or regular), by gender and by race.

The focus of the study was Level I of Hands-On Equations which constituted the first seven lessons of the program. These lessons, and the testing used in the study, included examples such as  $4x + 3 = 3x + 6$  and  $2(2x + 1) = 2x + 6$ . Based on prior research results with many classrooms around the country, including inner city classrooms, it was expected that the students at each of the grade levels would show significant pre- to post-test gains, and that the various grade levels would have similar post-test results notwithstanding initial pre-test difference. In addition, this study sought to investigate the effect on achievement due to sex, race and student classification.

It was understood that if the students were successful with the types of equations noted above, they would have overcome one of the major obstacles to the learning of algebra, namely the ability to understand and work with algebraic linear equations.

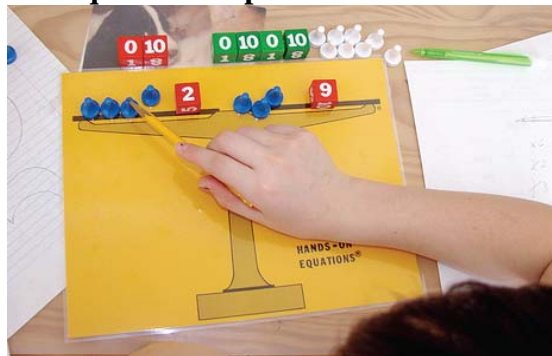
The teachers who participated in this study received a full day of training in the use of the program, broken up into two half-day after-school sessions. The workshop they attended, the Making Algebra Child's Play<sup>®</sup> workshop, was conducted by a certified Borenson and Associates, Inc. instructor in the spring of 2008. Immediately after instruction, the teachers administered a pre-test to their students, and then proceeded to teach the first six lessons of Hands-On Equations. They administered a post-test after Lesson #6 using the game pieces, taught Lesson #7, and administered a second post-test after Lesson #7 in which the students did not use the game pieces.

## GENERAL INTRODUCTION

### HANDS-ON EQUATIONS®

Hands-On Equations (HOE), is a program developed by Dr. Henry Borenson (one of the authors of the present study), to provide an intuitive, hands-on approach, to presenting algebraic concepts to grade school and middle school students. The program uses numbered-cubes to represent the constants, and blue pawns to represent the variable  $x$ . It also uses a scale representation on which the students “set up” the equation. A typical setup is shown in Figure 1.

#### Hands-On Equations Representation of $4x + 2 = 3x + 9$



**Figure 1**

The students then proceed to use “legal moves,” which are the mathematical counterpart of the abstract algebraic methods which are used to solve these linear equations. The system thus makes abstract linear equations visual and understandable, and further provides students with the means of solution through a kinesthetic approach which makes sense to them. An example of this approach is shown in Appendix 1.

The program is unique in that the abstract knowledge base needed by students to solve these equations is transformed into an easily understood and manageable set of verbal, visual and kinesthetic responses using manipulatives. The program teaches algebraic principles which students in grades 3 to 8 can apply in any sequence desired to solve the given equation. Hence, the students using Hands-On Equations need not memorize a series of steps to solve an equation, as is the case in more traditional instruction. Rather they feel empowered to use their thinking and understanding of basic principles to solve the problem at hand. (See Appendix 2 for the objectives of Level I of the program)

### META-ANALYSIS APPROACH

The research studies mentioned in this report, as well as the series of studies of which this is a part, use a multi-site replications design and a meta-analysis procedure to study the effect of the HOE program on many groups of students with different characteristics (regular education students, special education students, elementary, middle, and high school students, inner city, rural, suburban, gifted and handicapped). Some of the above groups will be studied separately. In other cases, the classroom with a diverse student population will be studied as a unit. Similar groups will then be combined into a larger study, thus the meta-analysis component of this design. As of the date of this report, we have data on more than 130 classrooms in 19 states involving over 2,500 students.

## DEFINITION OF TERMS

The definitions below will clarify the nature of each of the tests, as well as the various terms used in this report.

**Blue Pawn:** The student game pieces which are used to represent the variable  $x$ .  $4x$  for example, would be represented by 4 blue pawns. (See Fig. 1 which is on the previous page).

**Red Numbered Cubes:** The student game pieces used to represent the positive constants. The expression  $4x + 2$  would be represented by 4 blue pawns and a cube with the number 2 displayed. (See Fig. 1)

**Flat Laminated Balance:** A representation of a balance scale printed on paper, laminated for protection. The students set up their equation on the flat balance scale. (See Fig. 1)

**Game Pieces:** When students solve the equations using the manipulatives that come with the program, namely, the blue pawns, the number cubes and the flat laminated balance, we will say that “the students solved the equations using the game pieces.”

**Teacher’s Balance Scale and Game Pieces:** A stationary balance scale and game pieces used by the instructor in the front of the room to illustrate the equations.

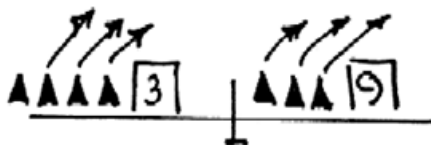


**Setup:** The set of all the pieces that are placed either on the student balance scale or on the Teacher’s Demonstration Scale to represent the algebraic equation.

**Legal Move:** These are the moves a student may perform and still keep the equation in balance. In Level I of HOE, which is the subject of the present study, the legal move is the Subtraction Property of Equality. In particular, the students may subtract the same number of pawns from both sides of the scale, or they may subtract the same cube value from both sides of the scale. Referring back to Figure 1 on the previous page, the students may subtract three blue pawns from each side of the setup to obtain the result from what is left. If they wish, they may also take away a 2 value from the cubes on both sides.

**Kinesthetic:** This term is used to indicate that the simplification of the equation is carried out in a physical manner through the use of bodily motions or gestures. For example, in simultaneously removing a pawn from each side of the balanced system, the student is developing a bodily sense of the mathematical principle known as the Subtraction Property of Equality. This bodily action, coupled with the use of the distinct game pieces to represent the various elements of the equation, along with the student verbalization of the process, leads to an in-depth multi-faceted understanding of the algebraic concepts. In addition, the bodily memory often serves to remind a student of the processes that they may be used to solve the equation. Hence, the term kinesthetic indicates the use of the additional learning modality which involves bodily memory.

**Pictorial Notation:** Once the students have learned to solve the equations with the game pieces, they learn to solve the equations using only paper and pencil by drawing pictures of the game pieces, the balance scale and the legal moves. The pictorial representation and solution to  $4x + 3 = 3x + 9$  is shown below and described in more detail in Appendix 3.



## DESCRIPTION OF TESTS USED IN THE STUDY

Each of the tests used in this study had six questions of increasing difficulty. Each of the questions on each of the tests was randomly selected from a pool of similar questions, each designed to test the student's ability to solve a particular type of equation. The test items were non multiple-choice. The students were to find the value for  $x$  and the value for the check. The scoring of each question, however, was only based on the value for  $x$ . The students were provided with 15 minutes to respond to each of the tests. Samples of all of these tests are provided in Appendix 4. Below is a description of the various tests used in this study, as well as the abbreviations that will sometimes be used to refer to them.

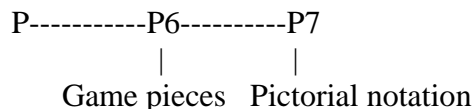
P: Pre-test. A test administered to students prior to their introduction to HOE.

P6: Lesson #6 Post-Test. A post-test in which the students were free to use the game pieces, flat laminated balance, and the methods learned in Lessons #1 - #6 of HOE to solve the equations.

P7: Lesson #7 Post-Test. A post-test in which the students were not allowed to use their game pieces, but could use the pictorial notation learned in Lesson #7 along with the concepts learned in the first six lessons of HOE. This post-test was used to determine the extent to which the students were able to move away from the use of the game pieces for solving equations to the pictorial system using only paper and pencil.

## PRIOR STUDIES

In order to measure the effectiveness of the first seven lessons of Hands-On Equations (Level I), studies were undertaken in which students were provided with a pre-test prior to instruction, followed by instruction on the first six lessons of the program and a post-test following Lesson #6. On this post-test, the students could use the game pieces to solve the problems. The students were then provided with Lesson #7, followed by a Lesson #7 post-test. On this post-test, the students were not allowed to use the game pieces; they could however use the pictorial notation learned in the program. This study model is shown schematically below:



The results of studies 59a (123 4<sup>th</sup> graders), 102b (196 6<sup>th</sup> graders) and 105a (105 8<sup>th</sup> graders) are summarized below. It is noted that for each group, the gain from the pre-test to each of these post-tests was statistically significant, and the gain was maintained or increased slightly as the students moved away from using the game pieces to the pictorial notation.

	<b>Pre-test</b>	<b>Post-test after Lesson #6 with game pieces</b>	<b>Post-test after Lesson #7 without game pieces</b>
<b>Grade 4, n=123 Study #59a</b>	<b>30%</b> (m=1.81)	<b>84%</b> (m=5.04) t(P, P6)= 22.62	<b>88%</b> (m=5.32) t(P, P7)=29.70
<b>Grade 6, n=190 Study #102b</b>	<b>48.2%</b> (m=2.89)	<b>92%</b> (m=5.54) t(P, P6)= 25.15	<b>93%</b> (m=5.64) t(P, P7)=22.48
<b>Grade 8, n=105 Study #105a</b>	<b>64.8%</b> (m=3.89)	<b>87.7%</b> (m=5.26) t(P, P8)=8.895	<b>88.8%</b> (m=5.34) t(P, P8)=9.99

We also note that the 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> grade students achieved at comparable levels on each of the post-test tests, notwithstanding the large disparity in the pre-test scores. Indeed, the pre-test scores ranged from 30% for the 4<sup>th</sup> graders to 64.8% for the 8<sup>th</sup> graders. Yet, the post-test results for all three grade groupings differed by less than 10% for the Lesson #6 post-test and by less than 5% for the Lesson #7 post-test. With all grade groupings, each of the post-test score was 84% or higher.

Studies conducted with 4<sup>th</sup> and 5<sup>th</sup> grade regular classrooms of the Broward County Public Schools, shown in the first two rows of the table below, yielded similar results. Both the Lesson #6 post-test scores and the Lesson #7 post-test scores differed by less than 5% and all scores exceeded 84%. We note that the group of gifted 5<sup>th</sup> grade students, shown in the third row below, began with a much larger pre-test score (more than double the pre-test scores of the other groups) yet attained post-test scores within 15% of the other groups. (Although these studies had a retention test which was also administered three weeks after the Lesson #7 post-test, the pre-test, the post-test after Lesson #6 and the post-test after Lesson #7 were identical to the studies cited in the previous paragraph.)

	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>Grade 4, n=111</b> <b>Study #131MA</b> <b>Regular students</b>	<b>26.8%</b> (m=1.61)	<b>84.2%</b> (m=5.05) t(P, P6)=20.50	<b>84.2%</b> (m=5.05) t(P, P7)=20.45
<b>Grade 5, n=84</b> <b>Study #138MA</b> <b>Regular students</b>	<b>37.7%</b> (m=2.26)	<b>88.3%</b> (m=5.30) t(P, P6)= 19.62	<b>88.5%</b> (m=5.31) t(P, P7)=17.09
<b>Grade 5, n=111</b> <b>Study #139MA</b> <b>Gifted Students</b>	<b>78%</b> (m=4.68)	<b>95.3%</b> (m=5.72)	<b>95.3%</b> (m=5.72)

#### RATIONALE OF THE STUDY

Prior studies suggested that students in grades 4 to 8 achieve at comparable levels, notwithstanding their grade level or their pre-test results. This study sought to expand this investigation to include the category of group classification (ELL, LD, or regular), gender classification, and race classification (Hispanic, Black, and White).

#### PURPOSE OF THE STUDY

The purpose of the current 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grade studies was five-fold, as noted below, with the last three being those unique to this study.

- a) First we wished to ascertain that for each of the grade levels the gain from the pre-test to each of the post-tests would be statistically significant; additionally, we wished to verify that at no grade level would the score decrease significantly in moving from Post-test Lesson #6 using the game pieces to Post-test Lesson #7 without the use of the game pieces.
- b) Secondly, we wished to ascertain that there would be no significant difference in achievement on either of the post-tests based on grade level.
- c) Thirdly, we wanted to see if there would be a significant difference based on gender on either of the post-tests for all the classes combined.
- d) Fourth, we wanted to see if there would be a significant difference in achievement based on group categorization, such as LD, ELL or Regular students.
- e) Fifth, we wanted to see if there would be a significant difference in achievement based on race for all the classes combined.

## TEACHERS OF THE STUDY

The teachers participating in this study were those who accepted an invitation by the mathematics coach to receive HOE training on condition that they conduct the research study with at least one of their classes.

None of the teachers in this study were new or beginning teachers. They were equally divided among teachers who had 1 – 3 years of teaching experience, 3 – 5 years, and 5 -1 0 years. For each of these teachers this was their first experience teaching HOE. Eleven teachers participated in the study and all indicated that they had taught the program as instructed and had not made any changes to the teaching procedures. (The data submitted by one teacher could not be used, for reasons noted in the appendix, but the results for that class are included there.)

## STUDENTS OF THE STUDY

The 243 students in this study were 65 6<sup>th</sup> graders, 102 7<sup>th</sup> graders and 76 8<sup>th</sup> graders. All of the teachers described their students as inner-city students. The population included 136 Regular, 22 ELL students, 49 LD and 27GT students. There were a total of 111 females and 132 males. According to the math coach, by state standards they are considered low achieving; however by district standards they are considered average.

## CLASSES OF THE STUDY

All of the summary forms submitted by ten of the teachers could be used for the study. The composition of each class by student classification is noted in Table 1. We note that the 8<sup>th</sup> grade population included a class of 22ELL student and a class of 21 LD students, as well as a class or regular students. Hence, the 8<sup>th</sup> grade would provide an opportunity to compare class performance by special group classification.

6 <sup>th</sup> Grade Classes	
#153	28R
#157	15LD
#158	22R

7 <sup>th</sup> Grade Classes	
#150a	27R
#150b	22R
#151	17R
#160	10LD
#150c	26GT

8 <sup>th</sup> Grade Classes	
#154	22R
#155	3LD, 8R
#156	22ELL
#159	21LD

**Table 1: Composition of classes by student classification**



## CLASSROOM INSTRUCTION USED IN THE STUDY

The teachers were to present each of the first seven lessons of the HOE program as instructed in the training seminar. Each lesson involved the teacher presenting a concept to the class, along with two or three practice examples. The time required for this instructional component varied among the classes, with some teachers requiring 12 minutes and others requiring 40 minutes. The average for the lesson presentation for this group of nine teachers was about 22 minutes. Following this learning session, the students were provided with a worksheet to complete. Four of the examples on the worksheet were on the current lesson; six of the examples reviewed concepts learned in prior lessons. The worksheets were specifically designed in this manner so that the students would be reviewing all prior lessons each time they did a worksheet. The time spent on the worksheets varied from 15 minutes to 32 minutes. The average time spent on the worksheets for these eleven classes was about 22 minutes.

For the first six lessons of the program, the teacher used the Teacher's Demonstration Scale and Teacher Game Pieces to illustrate the equations and concepts. The students used their sets of game pieces and their flat laminated balance for these lessons. In these first six lessons, both the teacher and the student used hand-gestures to carry out their legal moves. For Lesson #7, the teacher illustrated the pictorial solutions on the blackboard or overhead projector, and the students presented their solutions on paper at their desks. The worksheet for Lesson #7 contained four pictorial examples and six review examples using the game pieces. In the 7<sup>th</sup> lesson, the legal move was performed by drawing arrows to indicate the items to be removed.

## TESTING PROCEDURE

A pre-test was given to the students before they were exposed to the HOE program. At the conclusion of Lesson #6, the students were provided with a post-test in which they were at liberty to use their game pieces (the pawns, cubes, and laminated scale). The students were then taught Lesson #7, and given a second (different) post-test. This time the students were to take the post-test without using the game pieces. The students, however, were free to use the pictorial notation they had learned in Lesson #7.

All of the classes were taught by teachers who had participated in a one-day Making Algebra Child's Play<sup>®</sup> workshop conducted by a certified Borenson and Associates, Inc. instructor. This workshop was divided into two after school sessions which took place on April 3 and April 4, 2008. The teachers started teaching HOE to their students almost immediately after the training (pre-test given to individual classrooms between April 7 and April 8, 2008). The first six lessons were taught and the Lesson #6 post-test with the game pieces was administered between April 16 and April 30, 2008. The seventh lesson was then presented to the students and the Lesson #7 post-test was administered between April 16<sup>th</sup> and May 3<sup>rd</sup>, 2008.

Each classroom was analyzed as a unit, whether the students in the class were LD, ELL, GT, regular students, or a mixture of these groups. For the grade comparison study, the 7<sup>th</sup> grade gifted class was omitted, as this class scored very high on the post-tests (100%) and the other grade levels did not have a gifted class to balance it. For the gender study, this gifted class was included as the number of students was pretty evenly divided among boys and girls (12B, 18G).

## STUDY HYPOTHESES:

Note: H6 – H9 are new areas of investigation unique to this study.

**H1:** Pre-test average is expected to be higher by grade level, i.e., the 8<sup>th</sup> graders are expected to do better than the 7<sup>th</sup> graders who are expected to do better than the 6<sup>th</sup> graders

**H2:** At each grade level and for all the classes combined, the gain from pre-test to each of the post-tests will be statistically significant. Students at each grade level will be expected to achieve in the 85% range on each of the post-tests.

**H3:** For each class, grade level and for all the classes combined, students will maintain their Lesson #6 post test scores as they move to post-test for Lesson #7: There will be no significant decrease in scores. All prior studies showed either no difference or showed a significant increase in going to the pictorial notation.

**H4:** Students at all grade levels will attain comparable results on the post-test following Lesson #6, i.e. there will be no significant difference by grade level.

**H5:** Students at all grade levels will attain comparable results on the post-test following Lesson #7, i.e., there will be no significant difference by grade level.

**H6:** The post-test results will be similar for boys and girls for the combined classes as a whole on each of the post tests, with each group showing a significant increase from the pre-test to each of the post-tests

**H7:** At the 8<sup>th</sup> grade level, the LD and ELL classes are expected to achieve less on the pre-test than the students in the regular class

**H8:** At the 8<sup>th</sup> grade level, no difference in results is expected between the regular class, the LD class and the ELL class

**H9:** The result on each of the post-tests will be similar for Black, Hispanic and White students for all the classes combined

## RESULTS

Twelve classrooms were included in this study. Each classroom's data was analyzed independently to provide feedback to each teacher about the performance of their students. T-tests were conducted between the mean of the pre-test and the mean of each of the two post-tests for each of these 12 classes. For all but one of the 12 individual classes in this study, the gain from the pre-test to each of the post-tests was statistically significant. The one study in which the gain was not statistically significant was study #159 which had only eleven students. For that study the pre-test score was 69.7%, and the post-tests were 92.5% and 90.8%. However, the low number of students did not result in a statistically significant gain at the .01 level.

In three of the classes the score more than doubled from the pre-test to each of the post-tests. In percentage terms, the pre-test score varied among the 12 classes from a low of 28.3% for a 7<sup>th</sup> grade class of LD students (Study 160) to 83.3% for a class of gifted 7<sup>th</sup> graders (Study #150c). The Lesson #6 post-test varied from 69% for a class of regular 7<sup>th</sup> grade students (Study #150b) to 100% for the 7<sup>th</sup> grade class of gifted students (Study 150c). The Lesson #7 post-test varied from 71.7% for the 7<sup>th</sup> grade class of LD students (Study #160) to 100% for the class of 7<sup>th</sup> grade gifted students (Study #150c).

<b>6<sup>th</sup> Grade</b>		Pre-Test	Lesson #6 Post-Test	Lesson #7 Post-Test
#153	28R	<b>70.8%</b> M=4.25	<b>90.5%</b> M=5.43, t(P,P6)=4.67	<b>86.8%</b> M=5.21, t(P,P7)=5.30
#157	15LD	<b>48.8%</b> M=2.93	<b>88.8%</b> M=5.33, t(P,P6)=7.16	<b>88.8%</b> M=5.33, t(P,P7)=5.83
#158	22R	<b>55.3%</b> M=3.32	<b>90.2%</b> M=5.41, t(P,P6)= 7.50	<b>99.2%</b> M=5.95, t(P,P7)= 12.98
<b>7<sup>th</sup> grade</b>				
#150a	27R	<b>64.2%</b> M=4.48	<b>92%</b> M=5.52, t(P,P6)=3.85	<b>96.8%</b> M=5.81, t(P,P7)=5.89
#150b	22R	<b>46.2%</b> M=2.77	<b>69%</b> M=4.14, t(P,P6)=6.38	<b>90.2%</b> M=5.41, t(P,P7)=8.09
#151	17R	<b>50%</b> M=3.0	<b>88.4%</b> M=4.47, t(P,P6)=5.39	<b>87.3%</b> M=5.24, t(P,P7)=6.06
#160	10LD	<b>28.3%</b> M=1.7	<b>71.7%</b> M=4.30, t(P,P6)=3.70	<b>71.7%</b> M=4.30, t(P,P7)=3.98
#150c	26GT	<b>83.3%</b> M=5.0	<b>100%</b> M=6.0, t(P,P6)=4.51	<b>100%</b> M=6.0, t(P,P7)=4.51
<b>8<sup>th</sup> Grade</b>				
#154	22R	<b>52.3%</b> M=3.14	<b>85.7%</b> M=5.14, t(P,P6)=3.80	<b>89.3%</b> M=5.36, t(P,P7)=4.79
#155	3LD, 8R	<b>69.7%</b> M=4.18	<b>92.5%</b> M=5.55, t(P,P6)=2.68	<b>90.8%</b> M=5.45, t(P,P7)=2.28
#156	22ELL	<b>40.8%</b> M=2.45	<b>86.3%</b> M=5.18, t(P,P6)=8.10	<b>92.5%</b> M=5.55, t(P,P7)=10.81
#159	21LD	<b>40.5%</b> M=2.43	<b>89.7%</b> M=5.38, t(P,P6)=11.25	<b>85.7%</b> M=5.4, t(P,P7)=8.18

**Table 2: Class results on each of the tests. For the post-test after Lesson #6 students could use the game pieces. For the post-test after Lesson #7 the students did not use the game pieces. All t-values are significant except for those of study #155 due to the small n.**

For each of the 12 classes, a t-test was conducted between the mean of the post-test following Lesson #6, in which the students used the game pieces, and the mean of the post-test following Lesson #7, in which the students did not use the game pieces to see if there was a significant decline in score in moving away from the game pieces. Only one of the 12 studies, namely #153 showed a decline at all, and that decline was not significant (90.5% to 86.8%). In all the other studies, the students either maintained or increased their Lesson #6 post-test score on the Lesson #7 post-test. This increase was significant for study #150b, where the students did significantly better on the post-test following Lesson #7 than they did on the post-test following Lesson #6 (90.2% vs. 69%).

In order to obtain figures that would enable us to compare achievement by grade level, we omitted Study 150C, since this was a class of gifted students who scored very high on the post-tests, and such a class (which could be admitted into the study) existed only in the 7<sup>th</sup> grade. On the other hand, we did include the LD classes, since there was one at each grade level. Although there was only one ELL class, namely in the 8<sup>th</sup> grade, we did include that class since the post-test results from this class was very similar to those of the other classes. The low pre-test results for that class did, however, lower the pre-test mean for the whole group of 8<sup>th</sup> grade students.

Excluding the 7<sup>th</sup> grade class of gifted students (Study 150c), the group results are shown below. Since for each of the individual classes comprising these groups, except for that of Study #155, a significant t was found in comparing the pre-test to each of the post-tests, it was clear that for each grade level as well the gain from the pre-test to each of the post-tests was statistically significant. There was no need to conduct a t-test to ascertain this, since the t value would only increase in size with the larger n value.

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>Grade 6, n=65</b>	<b>60.5%</b> M=3.63	<b>90%</b> M=5.4	<b>91.3%</b> M=5.48
<b>Grade 7, n=76</b>	<b>54.5%</b> M=3.27	<b>78.7%</b> M=4.72	<b>89.5%</b> M=5.37
<b>Grade 8, n=76</b>	<b>48.2%</b> M=2.89	<b>87.8%</b> M=5.27	<b>90.5%</b> M=5.43
<b>School, n=217</b>	<b>54%</b> M=3.24	<b>85.3%</b> M=5.12	<b>90.3%</b> M=5.42

**Table 3: Achievement by grade level (gifted 7<sup>th</sup> grade class not included above)**

In order to assess the difference in performance among regular, LD and ELL students in the 8<sup>th</sup> grade, two classes were grouped together to form the regular 8<sup>th</sup> grade group. One of these classes had 22 regular students the other class had 8 regular and 3 LD students. Nonetheless, we included all 33 students in the regular category. This group of students was then compared with the LD self-contained class and with the ELL self-contained class. The results are shown in the table below.

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>8<sup>th</sup> Regular Class, n=33</b>	<b>58%</b> M=3.48	<b>88%</b> M=5.28	<b>89.8%</b> M=5.39
<b>8<sup>th</sup> ELL Class, n =22</b>	<b>40.8%</b> M=2.45	<b>86.3%</b> M=5.18	<b>92.5%</b> M=5.55
<b>8<sup>th</sup> LD Class n=21</b>	<b>40.5%</b> M=2.43	<b>89.7%</b> M=5.38	<b>90%</b> M=5.4
<b>8<sup>th</sup> Grade Combine n=76</b>	<b>48.2%</b> M=2.89	<b>87.8%</b> M=5.27	<b>90.5%</b> M=5.43

**Table 4: A comparison of achievement among 8<sup>th</sup> grade regular, LD and ELL students**

In order to compare the achievement by gender, all the males in all the 12 classes were combined and all the females in all the 12 classes were combined. The students in the gifted class were also included in this analysis since they were about equally divided into boys and girls. The results are shown in the table below.

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>Female Students n=111</b>	<b>60.3%</b> M=3.62	<b>88.7%</b> M=5.32, t(P,P6)=11.15	<b>92.7%</b> M= 5.56, t(P,P7)=14.29
<b>Male Students n =132</b>	<b>60.2%</b> M=3.61	<b>88.5%</b> M=5.31, t(P,P6)=11.3	<b>91.3%</b> M=5.48, t(P,P7)= 12.45

**Table 5: An analysis of student achievement by gender; the 7<sup>th</sup> grade gifted class also included.**

## CONCLUSIONS

This study leads to the following conclusions:

**Hypothesis1:** Pre-test average is expected to be higher by grade level, i.e., the 8<sup>th</sup> graders are expected to do better than the 7<sup>th</sup> graders who are expected to do better than the 6<sup>th</sup> graders

Based upon prior studies, it was expected that students further along in their schooling would achieve a higher score on their pre-test than students in earlier grades. H1 was not substantiated

in the current study. In fact, the 6<sup>th</sup> grade students had the highest pre-test score (60.5%), followed by the 7<sup>th</sup> grade (54.5%) and finally by the 8<sup>th</sup> grade (48.2%). The low pre-test score obtained by the 8<sup>th</sup> grade is attributable in part to that group containing an ELL class which had a pre-test average of 40.5%. None of the other groups had an ELL class. The high pre-test score for the 6<sup>th</sup> grade was largely the contribution of the class in Study # 153 which had a pre-test average of 70.8%. Although H1 was not substantiated in this study, this result has been noted in other studies (see page 4 of this report). Hence, it seems reasonable to propose that students higher in their educational ladder in grades 4 – 8 will tend to do better on the pre-test due to having greater exposure to mathematics than students lower in grade level.

**H2:** For each grade level and for all the classes combined, the gain from pre-test to each of the post-tests will be statistically significant. Students at each grade level will be expected to achieve in the 85% range on each of the post-tests.

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>Grade 6, n=65</b>	<b>60.5%</b> M=3.63	<b>90%</b> M=5.4	<b>91.3%</b> M=5.48
<b>Grade 7, n=76</b>	<b>54.5%</b> M=3.27	<b>78.7%</b> M=4.72	<b>89.5%</b> M=5.37
<b>Grade 8, n=76</b>	<b>48.2%</b> M=2.89	<b>87.8%</b> M=5.27	<b>90.5%</b> M=5.43
<b>Combined, n=217</b> (excluding 7 <sup>th</sup> grade GT class)	<b>54%</b> M=3.24	<b>85.3%</b> M=5.12	<b>90.3%</b> M=5.42
<b>7<sup>th</sup> grade GT Class</b> <b>n=26</b> (Study 150c)	<b>83.3%</b> M=5.0	<b>100%</b> M=6.00, t(P,P6)=4.51	<b>100.3%</b> M=6.00, t(P,P7)=4.51

**Table 3: Achievement by grade level**

With the exception of the class of Study 155, for which the value of n was too small to yield a significant value for t, the gain from the pre-test to each of the post-tests was statistically significant for each of the individual classes,. Hence, there was no need to conduct a group t-test to verify that for each of the grade groups, this gain was also statistically significant. The 7<sup>th</sup> grade group obtained 78.7% on the post-test following Lesson #6. Other than this case all of the groups attained 85% or more on each of the post-tests. H2 is thus confirmed.

**H3:** For each class, grade level and for all the classes combined, students will maintain their Lesson #6 post test scores as they move to post-test for Lesson #7: There will be no significant decrease in scores. All prior studies showed either no difference or showed a significant increase in going to the pictorial notation.

Except for Study #153 (the score decreased from 90.5% to 86.8%), the students maintained their Lesson #6 post-test result or increased their score in moving from the post-test following Lesson

#6 using the game pieces to the post-test following Lesson #7 without the use of the game pieces. In one study, namely #150b, the increase was statistically significant. H3 is confirmed.

**H4:** Students at all grade levels will attain comparable results on post-test 6, i.e. there will be no significant difference by grade level.

The 6<sup>th</sup> and 8<sup>th</sup> grades attained comparable results on the post-test following Lesson #6, namely 90% and 87.8%. The 7<sup>th</sup> grade group attained only 78.7% on this post-test. This low result is attributable mostly to the class of study 150b which scored 69% on this test. Nonetheless, H4 is confirmed since the difference is not statistically significant.

**H5:** Students at all grade levels will attain comparable results on post-test 7, i.e., there will be no significant difference by grade level

The scores on the post-test following Lesson #7, in which the students did not use the game pieces, differed by less than 2% among the various grade groups, varying from 89.5 to 91.3%. H5 is confirmed.

**H6:** The results will be similar for boys and girls for the combined classes as a whole on each of the post tests, with each gender group showing a significant increase from the pre-test to each of the post-tests

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>Female Students n=111</b>	<b>60.3%</b> M=3.62	<b>88.7%</b> M=5.32, t(P,P6)=11.15	<b>92.7%</b> M= 5.56, t(P,P7)=14.29
<b>Male Students n =132</b>	<b>60.2%</b> M=3.61	<b>88.5%</b> M=5.31, t(P,P6)=11.3	<b>91.3%</b> M=5.48, t(P,P7)= 12.45

**Table 5: An analysis of student achievement by gender; the 7<sup>th</sup> grade gifted class is included.**

The score obtained by the group of 111 females and 132 boys were practically identical on the pre-test as well as on each of the post-tests, differing by less than half of one percent on post-test following Lesson #6 (88.7% and 88.5%) and less than 2% on the post-test following Lesson #7 (92.7% and 91.3%). All the t-values measuring the gain from the pre-test (60.3% and 60.2%) to each of the post-test for both gender groups indicated that the gain was statistically significant (t was greater than 10). In these classes the boys and girls were integrated. H6 is confirmed. Note: Study 150C, the gifted 7<sup>th</sup> grade class was included in this analysis since that class was about equally divided between boys and girls.

**H7:** At the 8<sup>th</sup> grade level, the LD and ELL classes are expected to achieve less on the pre-tests than the students in the regular class

The 8<sup>th</sup> grade LD class attained 40.5% on the pre-test; the 8<sup>th</sup> grade ELL class attained 40.8% on the pre-test. The 8<sup>th</sup> grade regular class attained 58% on the pre-test. In percentage terms, the LD and ELL class did about the same and each of them attained about 30% less than the regular class on the pre-test. H7 is confirmed.

**H8:** At the 8<sup>th</sup> grade level, no difference in results is expected between the regular class, the LD class and the ELL class

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>8<sup>th</sup> Regular Class, n=33</b>	<b>58%</b> M=3.48	<b>88%</b> M=5.28	<b>89.8%</b> M=5.39
<b>8<sup>th</sup> ELL Class, n =22</b>	<b>40.8%</b> M=2.45	<b>86.3%</b> M=5.18	<b>92.5%</b> M=5.55
<b>8<sup>th</sup> LD Class n=21</b>	<b>40.5%</b> M=2.43	<b>89.7%</b> M=5.38	<b>90%</b> M=5.4
<b>8<sup>th</sup> Grade Combine n=76</b>	<b>48.2%</b> M=2.89	<b>87.8%</b> M=5.27	<b>90.5%</b> M=5.43

**Table 4: A comparison of achievement among 8<sup>th</sup> grade regular, LD and ELL students**

The Lesson #6 post-test results varied from 86.3% to 89.7%, with a difference of less than 4% among the scores; the Lesson #7 post-tests varied from 89.8% to 92.5%, with a difference of less than 3% among the scores. Hence, each of the three groups of 8<sup>th</sup> grade classes, namely regular, LD and ELL attained comparable results on each of the post-tests. H8 is thus confirmed.

**H9:** The results on each of the post-tests will be similar for Black, Hispanic and White students for the school as a whole.

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>Latino n=126</b>	<b>57.8%</b> M=3.47	<b>87.8%</b> M=5.27	<b>92%</b> M=5.52
<b>Black n =27</b>	<b>51.8%</b> M=3.11	<b>83.3%</b> M=5.00	<b>91.3%</b> M=5.48
<b>White n=14</b>	<b>70.2%</b> M=4.21	<b>91.7%</b> M=5.50	<b>92.8%</b> M=5.57

The idea of gathering race data in this study arose after the printed documents had been provided to the teachers. The code for racial classification was communicated verbally; as a result the data notation used by the teachers was not consistent. Some teachers used “A” for



Asian, for examples, and others used “A” for African American. Of the 243 students, 167 entries were clearly distinguishable. The summary results for those entries are noted above. These results suggest that H9 is a hypothesis which should be maintained and researched further; the data that was received from the current study seems to suggest the proposition that students of various ethnic composition will do similarly well with Hands-On Equations, with or without the game pieces. However, since there were technical problems with the gathering of this data, we are not able to enter the above results into the evidentiary record.

**SUMMARY: Interim Report Dec. 01, 2008**  
**6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> Grade Inner City Students**

In summary, 243 inner city middle school students comprising twelve classes participated in this study. The results obtained confirm previous research results that a) students at all grade levels, from the 4<sup>th</sup> grade to the 8<sup>th</sup> grade, make statistically significant gains in moving from the pre-test to each of the post-tests, b) they achieve at comparable levels on the Lesson #6 post-test using the game pieces (80% or above), and c) the students maintain this achievement as they move to the Lesson #7 post-test using the pictorial notation (80% or above). The current study, additionally, showed that student achievement was not affected by gender, with males and females obtaining almost identical results on each of the post-tests, nor was it affected by special class grouping: LD, ELL and Regular students.

Regarding the gender study, each class was comprised of boys and girls. The boys in all the classes were combined into one group as were the girls. We note the almost identical pre-test scores obtained by the boys and by the girls. One might propose that this equal “starting line” may have been a factor in both sexes obtaining similar results on each of the post-tests. However, other results, such as that of grade grouping, or by special group classification, for example, demonstrate that even when the groups do not start at the same point, their achievement are at comparable levels.

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>Grade 6, n=65</b>	<b>60.5%</b> M=3.63	<b>90%</b> M=5.4	<b>91.3%</b> M=5.48
<b>Grade 7, n=76</b>	<b>54.5%</b> M=3.27	<b>78.7%</b> M=4.72	<b>89.5%</b> M=5.37
<b>Grade 8, n=76</b>	<b>48.2%</b> M=2.89	<b>87.8%</b> M=5.27	<b>90.5%</b> M=5.43
<b>School, n=217</b> (excluding GT class)	<b>54%</b> M=3.24	<b>85.3%</b> M=5.12	<b>90.3%</b> M=5.42

<b>7<sup>th</sup> grade GT Class</b> <b>n=26</b>	<b>83.3%</b> M=5.0	<b>100%</b> M=6.00, t(P,P6)=4.51	<b>100.3%</b> M=6.00, t(P,P7)=4.51
---	-----------------------	-------------------------------------	---------------------------------------

<b>Female Students</b> <b>n=111</b>	<b>60.3%</b> M=3.62	<b>88.7%</b> M=5.32, t(P,P6)=11.15	<b>92.7%</b> M= 5.56, t(P,P7)=14.29
<b>Male Students</b> <b>n =132</b>	<b>60.2%</b> M=3.61	<b>88.5%</b> M=5.31, t(P,P6)=11.3	<b>91.3%</b> M=5.48, t(P,P7)= 12.45

<b>8<sup>th</sup> Regular Class, n=33</b>	<b>58%</b> M=3.48	<b>88%</b> M=5.28	<b>89.8%</b> M=5.39
<b>8<sup>th</sup> ELL Class, n =22</b>	<b>40.8%</b> M=2.45	<b>86.3%</b> M=5.18	<b>92.5%</b> M=5.55
<b>8<sup>th</sup> LD Class n=21</b>	<b>40.5%</b> M=2.43	<b>89.7%</b> M=5.38	<b>90%</b> M=5.4

The special group classification showed that the 8<sup>th</sup> grade LD and ELL classes, even though starting with a pre-score 20% lower than the regular group, achieved at the same level on each of the post-tests. This result, as well as those related to grade grouping or gender, lead Larry Barber, one of the researchers of this study says that Hands-On Equations is an “equalizer,” meaning that any initial group differences (with the possible exception of grouping by giftedness) do not affect the ability of the students to achieve strong and comparable results (80% or above), even when the pre-test scores differ widely.

Note: Due to the manner in which the race data was collected, and the limited number of Black and White respondents, we are not able to make any definitive statement of the effect of race on student achievement, and that data is not being entered into the evidentiary record.

## **The Authors of the Study**

**Larry W. Barber** has served as past vice president of the American Educational Research Association. He has served as an assistant superintendent of schools and for 19 years was Director of Research for Phi Delta Kappa.

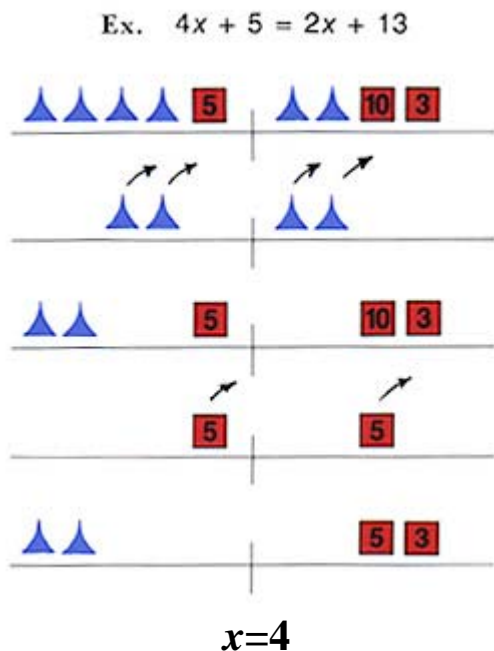
**Henry Borenson** received his doctorate in educational administration from Teachers College, Columbia University. His teaching experience includes teaching students in the South Bronx of New York City as well as students who represented the U.S. in the International Mathematical Olympiad while a teacher at Stuyvesant High School. Borenson received a U.S. patent for the Hands-On Equations teaching methodology. He is currently President of Borenson and Associates, Inc. Since 1990, more than 25,000 teachers have attended the Making Algebra Child's Play Workshop which he designed to help teachers obtain the maximum value from Hands-On Equations.

## References

- 1) Barber, L.W., Forbes, R.H., and Fortune, J.C. Developing Research Skills for Professional Educators (Participants Manual). Bloomington, IN: Phi Delta Kappa, 1988.
- 2) Barber, L.W., and Langdon, C., Academic Underachievement in the First Grade: Final Report of the Joint Study by Phi Delta Kappa and the Russian Academy of Education: Bloomington, IN: Phi Delta Kappa, 1994.
- 3) Barber, L.W., Carbo, M., and Thomasson, R., A Comparative Study of the Reading Styles Program to Extant Programs of Teaching Reading, Bloomington IN: Phi Delta Kappa, 1998.
- 4) Barber, L.W. and Borenson, H., Hands-On Equations Research Design and Interim Results: Dec. 2006: The Effect of the Making Algebra Child's Play Seminar on Teacher Self-Concept and Student Achievement, Borenson and Associates, Inc. 2006.
- 5) Barber, L.W. and Borenson, H., Hands-On Equations Interim Results, July 7, 2007 (Report #33c): The Effect of Hands-On Equations on the Learning of Algebra by 5<sup>th</sup> Grade Inner City Students, Borenson and Associates, Inc. 2007
- 6) Barber, L.W. and Borenson, H. Hands-On Equations, Interim Report, Nov 19, 2007: A Comparison of Algebra Achievement by 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> graders, Borenson and Associates Inc, 2007.
- 7) Barclay, J., A study of a Manipulative Approach to Teaching Linear Equations to Sixth Grade Students. (Masters Degree Dissertation), Texas Woman's University, Denton Texas, 1992.
- 8) Borenson, H. The Hands-On Equations Learning System. Borenson and Associates, Allentown, PA. 1986, 2009.
- 9) Borenson, H, and Barber, L.W. Hands-On Equations Interim Report: March 17, 2008: The Effect of Hands-On Equations on the Learning of Algebra by 4<sup>th</sup> and 5<sup>th</sup> graders of Broward County, Borenson and Associates, Inc., Allentown, PA 2008.
- 10) Dembo, Myron H., and Gibson, Sherry, Teacher's Sense of Efficacy: An Important Factor in School Improvement, The Elementary School Journal, Vol. 86, No. 2 (Nov. 1995), p. 173)
- 11) Edwards, A.L. Statistical Methods for the Behavioral Sciences, Holt Rinehart, and Winston, New York, 1963.
- 12) Frymier, J., Barber, L.W., et al Manual of Instruction for A Study of Students at Risk. Bloomington IN: Phi Delta Kappa, 1988.
- 13) Frymier, J., Barber L.W., et al "Simultaneous Replication: A Technique for Large Scale Research." Phi Delta Kappa, vol. 71, no. 3, November 1989, pp. 228-231.
- 14) Frymier, J., Robertson, N. and Barber, L.W., Learning to Fail: Case Studies of Students at Risk, 129 pages. Bloomington, IN. Phi Delta Kappa, 1991.
- 15) Frymier, J., Barber, L.W., et al., Growing up is Risky Business and Schools are not to Blame. 240 pages. Bloomington, IN: Phi Delta Kappa, 1992 a.
- 16) Frymier, J., Barber, L.W., et al., Assessing and Predicting Risk among Students in School, 351 pages. Bloomington. IN: Phi Delta Kappa, 1992 b.
- 17) Kearns, Sophie, The Relationship Between Teacher Efficacy, Educational Beliefs and Optimism Bias Among Preservice Teacher, paper presented at the Australian Association for Research in Education, 1995 Conference.
- 18) Slavin R.E., & Madden, N.A., (1993, April). Multi-site replicated experiments: An application to success for all. Paper presented at the annual meeting of the American Educational Research Association, Atlanta
- 19) University of Rochester (2007, July 28). Hand Gestures Dramatically Improve Learning. *ScienceDaily*. Retrieved December 4, 2008, from <http://www.sciencedaily.com/releases/2007/07/070725105957.htm>

## Appendix 1


The essential method used in Hands-On Equations is illustrated by the following example:



The  $x$ 's are represented by blue pawns and the constants by number cubes; the equation is “physically set” up on the laminated balance scale.

“Legal moves” are used to simplify the equation. In the 2<sup>nd</sup> step above, two blue pawns are removed from each side of the balance. The 3<sup>rd</sup> step indicates what is left. The 4<sup>th</sup> step shows the legal move of removing a 5 value from the cubes on both sides. In the last step the student recognizes that each pawn has a value of 4.

**Appendix 2**  
**HANDS-ON EQUATIONS® LEARNING SYTEM**  
**Teaching Objectives**

<b>Equation</b>	<b>Teaching Point/Objective</b>
<b>Level I, Lessons 1 – 7</b>	
Lesson #1 	In any specific problem, all the blue pawns have the same value, and the scale is in balance. <b>Solve for the pawn using <i>trial and error</i> and intuitive thinking.</b>
Lesson #2 $2x + x = x + 8$	The pawn has a special name, “ $x$ ”. <b>Transform the equation into its physical representation</b> , using the blue pawn for the $x$ and the red number cube for the number constant. The two sides of the equal sign become the two sides of the scale. Use trial and error to find the value of $x$ .
Lesson #3 $4x + 2 = 3x + 9$	The <i>legal move</i> with pawns is introduced: <b>we may remove the same number of blue pawns from both sides of a balanced system</b> (Subtraction Property of Equality).
Lesson #4 $4x + 5 = 2x + 13$	The legal move with the cubes is introduced: <b>We may subtract the same number cube or cube value from both sides of a balanced system</b> (Subtraction Property of Equality).
Lesson #5 $5x - 3x + 2 = x + 5$	<b>Students take away pawns as part of the setup process.</b> Distinguish the set up from a legal move, which comes <u>after</u> the setup has been completed.
Lesson #6 $2(x + 3) = x + 8$	<b>The students learn that the number outside the parenthesis indicates how many times the expression inside the parenthesis is set up on the balance scale.</b> (Some students learn the distributive law without being taught!)
Lesson #7 $4x + 3 = 3x + 9$	<b>Transfer the hands-on experiences of Level I to a pictorial system.</b> The $x$ 's are represented by shaded triangles, the number constants by boxed numbers, and the balance scale by a drawing of the scale. No plus signs are placed on the scale, only pawns or cubes.

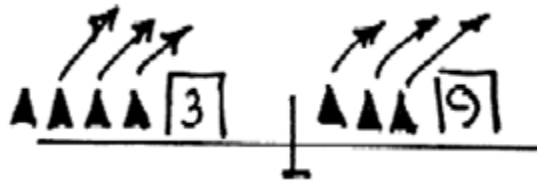
Copyright © Borenson and Associates, Inc. 2008

## Appendix 3

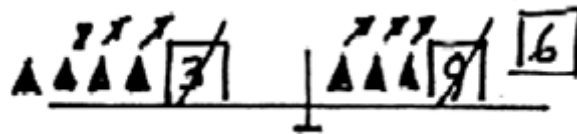
### PICTORIAL NOTATION

$$4x + 3 = 3x + 9$$

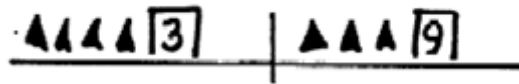
The  $x$ 's are represented by shaded triangles; the constants by boxed numbers. The equality of the two sides is indicated by the two sides of the balance scale.



Legal moves may be illustrated by erasing, crossing out, or using arrows. In the above example, most students will see that the pawn is worth 6. If the student wishes, he/she may cross off the 3 cube and replace the 9 cube with a 6 cube:



It is now clear that the pawn is worth 6. In order to conduct the check, the student redraws the original setup:



We see that the check is:  $27 = 27$



**Appendix 4**  
**TEST QUESTIONS FOR STUDY #131MA**

<p><b><u>Pre-Test Questions</u></b></p> <ol style="list-style-type: none"><li>1. <math>2x = 8</math></li><li>2. <math>x + 3 = 8</math></li><li>3. <math>2x + 1 = 13</math></li><li>4. <math>3x = x + 12</math></li><li>5. <math>4x + 3 = 3x + 6</math></li><li>6. <math>2(2x + 1) = 2x + 6</math></li></ol> <p><b><u>Post -Test After Lesson #6</u></b></p> <ol style="list-style-type: none"><li>1. <math>2x = 10</math></li><li>2. <math>x + 3 = 8</math></li><li>3. <math>2x + 2 = 10</math></li><li>4. <math>3x = x + 4</math></li><li>5. <math>4x + 3 = 3x + 9</math></li><li>6. <math>2(2x + 1) = 2x + 8</math></li></ol>	<p><b><u>Post-Test After Lesson #7</u></b></p> <ol style="list-style-type: none"><li>1. <math>2x = 6</math></li><li>2. <math>x + 3 = 10</math></li><li>3. <math>2x + 1 = 7</math></li><li>4. <math>3x = x + 2</math></li><li>5. <math>4x + 3 = 3x + 7</math></li><li>6. <math>2(2x + 1) = 2x + 10</math></li></ol> <p><b><u>Three-Week Retention Test</u></b></p> <ol style="list-style-type: none"><li>1. <math>2x = 4</math></li><li>2. <math>x + 3 = 13</math></li><li>3. <math>2x + 1 = 17</math></li><li>4. <math>3x = x + 14</math></li><li>5. <math>4x + 3 = 3x + 8</math></li><li>6. <math>2(2x + 3) = 2x + 10</math></li></ol>
---	---

## **Appendix 5**

### **Class not Included in the Study**

This study was excluded since the teacher noted 90 minutes for the instructional time for Lesson #1, as well as for several of the other lessons, which would seem to suggest unusual difficulty in teaching the concepts, and yet this class of gifted students achieved 90% on the pre-test! It is possible, according to the math coach, that this teacher simply noted the length of the class period, 45 minutes or 90 minutes, rather than the instructional time for each lesson. Since we were not able to definitely clarify this matter, we excluded this class from the study, including the gender study which did include the other gifted class.

<b>N= number of students</b>	<b>Pre-test</b>	<b>Post-test after Lesson #6</b>	<b>Post-test after Lesson #7</b>
<b>6<sup>th</sup> Grade GT n=30</b>	<b>90%</b> M=5.4	<b>98%</b> M=5.9	<b>100%</b> M=6.0

## Appendix 6

### Test Item Analysis # 158 MA (6th Grade Inner City) Studies 153, 157, 158 Test Item Analysis by Test Item Number

Pretest	Students	#1	#2	#3	#4	#5	#6
153	28	27	27	24	20	13	8
157	15	13	13	10	3	5	0
158	22	22	20	19	6	3	3
Totals	65	62	60	53	29	21	11
%	--	0.95	0.92	0.82	0.45	0.32	0.17

Post-test 6	Students	#1	#2	#3	#4	#5	#6
153	28	27	27	26	26	26	18
157	15	15	14	14	13	14	10
158	22	22	21	18	20	20	18
Totals	65	64	62	58	59	60	46
%	--	0.98	0.95	0.89	0.91	0.92	0.71

Post-test 7	Students	#1	#2	#3	#4	#5	#6
153	28	28	26	27	25	22	18
157	15	15	14	15	12	13	11
158	22	22	22	22	22	22	21
Totals	65	65	62	64	59	57	50
%	--	1.00	0.95	0.98	0.91	0.88	0.77

## Appendix 7

### Test Item Analysis # 160 MA (7th Grade Inner City) Studies 150A, 150B, 151, 160

Pre-test	Students	#1	#2	#3	#4	#5	#6
150A	27	27	24	26	16	16	12
150B	22	21	22	21	13	14	12
151	17	17	16	9	5	3	1
160	10	4	9	3	0	1	0
Totals	76	69	71	59	34	34	25
%	--	0.91	0.93	0.78	0.45	0.45	0.33

Post-test 6	Students	#1	#2	#3	#4	#5	#6
150A	27	27	26	25	26	26	19
150B	22	21	19	20	20	21	14
151	17	15	17	15	12	10	7
160	10	8	9	6	7	7	6
Totals	76	71	71	66	65	64	46
%	--	0.93	0.93	0.87	0.86	0.84	0.61

Post-test 7	Students	#1	#2	#3	#4	#5	#6
150A	27	26	27	25	26	27	26
150B	22	21	21	21	22	22	22
151	17	15	17	16	16	13	14
160	10	6	7	6	9	8	7
Totals	76	68	72	68	73	70	69
%	--	0.89	0.95	0.89	0.96	0.92	0.91

## Appendix 8

### Test Item Analysis # 159 MA (8th Grade Inner City) Studies 154, 155, 156, 159

Pretest	Students	#1	#2	#3	#4	#5	#6
154	22	14	15	12	9	11	8
155	11	10	10	10	5	7	4
156	22	21	18	12	1	1	1
159	21	18	15	14	0	2	2
Totals	76	63	58	48	15	21	15
%	--	0.83	0.76	0.63	0.20	0.28	0.20

Post-test 6	Students	#1	#2	#3	#4	#5	#6
154	22	21	19	21	20	17	15
155	11	10	11	10	11	9	10
156	22	22	22	20	20	19	11
159	21	20	20	20	21	17	15
Totals	76	73	72	71	72	62	51
%	--	0.96	0.95	0.93	0.95	0.82	0.67

Post-test 7	Students	#1	#2	#3	#4	#5	#6
154	22	22	22	19	20	17	18
155	11	11	11	11	11	10	6
156	22	22	22	22	20	21	15
159	21	19	17	17	19	18	18
Totals	76	74	72	69	70	66	57
%	--	0.97	0.95	0.91	0.92	0.87	0.75

## Appendix 9 Item Analysis

### Test Item Analysis # 150C MA (7th Grade Inner City Gifted Class) Study 150C

Pre-Test	Students	#1	#2	#3	#4	#5	#6
150C	26	26	26	25	21	17	15
Total	26	26	26	25	21	17	15
%	--	1.00	1.00	0.96	0.81	0.65	0.58

Post-Test 6	Students	#1	#2	#3	#4	#5	#6
150C	26	26	26	26	26	26	26
Total	26	26	26	26	26	26	26
%	--	1.00	1.00	1.00	1.00	1.00	1.00

Post-Test 7	Students	#1	#2	#3	#4	#5	#6
150C	26	26	26	26	26	26	26
Total	26	26	26	26	26	26	26
%	--	1.00	1.00	1.00	1.00	1.00	1.00

	Equation	Pre-test	Lesson #7 Post-Test	Equation
Question #1	$2x = 8$	100%	100%	$2x = 6$
Question #2	$x + 3 = 8$	100%	100%	$x + 3 = 10$
Question #3	$2x + 1 = 13$	96%	100%	$2x + 1 = 7$
Question #4	$3x = x + 12$	81%	100%	$3x = x + 2$
Question #5	$4x + 3 = 3x + 6$	65%	100%	$4x + 3 = 3x + 7$
Question #6	$2(2x+1) = 2x + 6$	58%	100%	$2(2x+1) = 2x + 10$

**This gifted 7<sup>th</sup> grade class was included in the gender analysis but not in the grade grouping analysis**

## Appendix 10 Item Analysis

Below, we show the percentage of students who obtained the item correct on the pre-test vs. the percentage of students who obtained the comparable item correct on the Lesson #7 post-test, without the use of the game pieces.

Grade 6, n =65. Study #158MA  
Percentage of Students with Correct Item Response

	Equation	Pre-test	Lesson #7 Post-Test	Equation
Question #1	$2x = 8$	95%	100%	$2x = 6$
Question #2	$x + 3 = 8$	92%	95%	$x + 3 = 10$
Question #3	$2x + 1 = 13$	82%	98%	$2x + 1 = 7$
Question #4	$3x = x + 12$	45%	91%	$3x = x + 2$
Question #5	$4x + 3 = 3x + 6$	32%	88%	$4x + 3 = 3x + 7$
Question #6	$2(2x+1) = 2x + 6$	11%	77%	$2(2x+1) = 2x + 10$

Grade 7, n =76. Study #160MA  
Percentage of Students with Correct Item Response

	Equation	Pre-test	Lesson #7 Post-Test	Equation
Question #1	$2x = 8$	91%	89%	$2x = 6$
Question #2	$x + 3 = 8$	93%	95%	$x + 3 = 10$
Question #3	$2x + 1 = 13$	78%	89%	$2x + 1 = 7$
Question #4	$3x = x + 12$	45%	96%	$3x = x + 2$
Question #5	$4x + 3 = 3x + 6$	45%	92%	$4x + 3 = 3x + 7$
Question #6	$2(2x+1) = 2x + 6$	33%	91%	$2(2x+1) = 2x + 10$

Grade 8, n =76. Study #159MA  
Percentage of Students with Correct Item Response

	Equation	Pre-test	Lesson #7 Post-Test	Equation
Question #1	$2x = 8$	83%	97%	$2x = 6$
Question #2	$x + 3 = 8$	76%	95%	$x + 3 = 10$
Question #3	$2x + 1 = 13$	63%	91%	$2x + 1 = 7$
Question #4	$3x = x + 12$	20%	92%	$3x = x + 2$
Question #5	$4x + 3 = 3x + 6$	28%	87%	$4x + 3 = 3x + 7$
Question #6	$2(2x+1) = 2x + 6$	20%	75%	$2(2x+1) = 2x + 10$