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Demystifying the Learning of Algebra Using Clear Language, Visual Icons, and Gestures

by Henry Borenson, E.D.

"We know from experience," according to the National Research Council (NRC), "that the current school approach to algebra is too abstract and an unmitigated disaster for most students" (NRC, 1998, 42). Many teachers of grades three through nine attend a Making Algebra Child's Play® workshop because they know firsthand that the traditional abstract approach does not work with far too many students. They are looking for something more visual, more hands-on, more intuitive, and yes, more fun.

A survey of 751 teachers conducted at the start of the morning session of several of these workshops revealed that only 16.2 percent of the teachers attending were confident of being able to teach significant algebraic concepts to more than 80 percent of their lowest achieving class using the traditional abstract approach. At the conclusion of the morning sessions, 98.4 percent of the attendees expressed confidence that they could do so by using the Hands-On Equations® methods that are demonstrated in these workshops (Barber and Borenson, 2006).

Since this survey, I have conducted classroom studies with researcher Larry Barber involving more than 2,700 students in 150 classes that have confirmed the workshop attendees' expectations. For example, in a study of 195 fourth- and fifth-grade students in Broward County, Florida, 83 percent successfully solved the equation $4x + 3 = 3x + 7$ *without* using the game pieces that are integral to the Hands-On Equations method described in this article but, instead, by using a subsequently taught pictorial notation (Borenson and Barber, 2008).

The Making Algebra Child's Play workshops empower teachers to demystify the learning of algebra by training them to use three essential teaching and learning components: clear language, visual icons, and gestures.

Clear Language. Throughout the training we emphasize the importance of correct mathematical language. For example, teachers using traditional instruction often try to help students solve algebraic equations by saying, "Whatever you do to one side, you must do to the other." Sound familiar? However, consider the student who tries to follow the teacher's instructions. Given the problem $4x + 3 = 3x + 9$ and not really knowing what the symbols mean, the student ends up doing this:

$$\mathbf{4}x + \cancel{\mathbf{3}} = \cancel{\mathbf{3}}x + \mathbf{9}$$

The student thinks he or she *has* done the same thing to both sides. In Hands-On Equations, we help students avoid this error by using more specific language and by visually clarifying the meaning of the constant from that of the coefficient, which I will demonstrate in this article.

Visual Icons. Using the Hands-On Equations methods, the equation $4x + 3 = 3x + 9$ is represented with specific visual icons, namely pawns, number cubes, and a representation of a balance scale (shown below). The x's are represented by blue pawns, the positive number constants by red number cubes.

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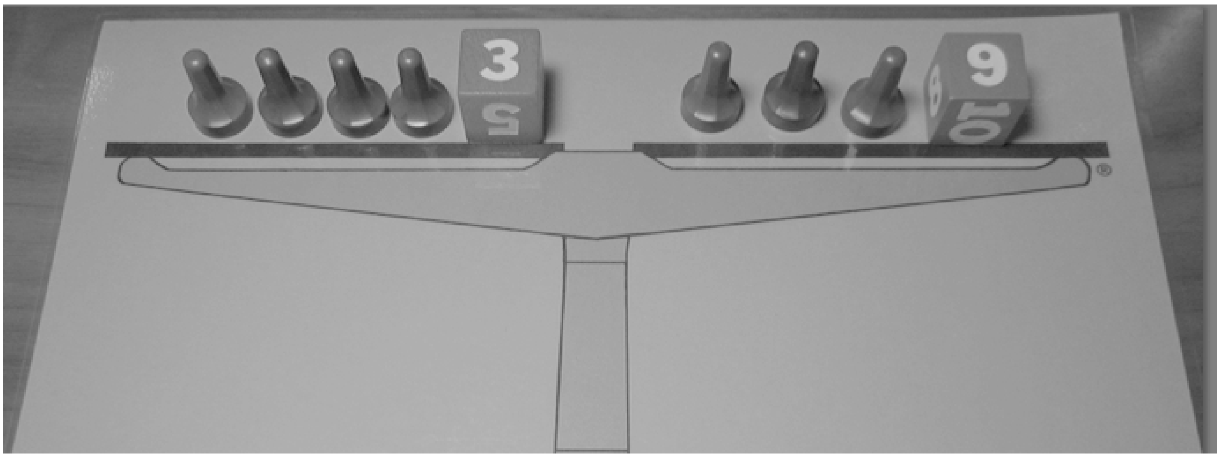

This visual representation of the equation makes it easy for students to distinguish the unknown from the constants and to understand the meaning of the equation. This physical setup demystifies the meaning of the abstract equation. Indeed, the student can now try to solve the equation using trial and error. For example, $x = 5$, does not work because the left side would be worth 23 while the right side would be worth 24.

Gestures. After the trial-and-error period, students learn an essential concept that enables them to simplify the setup. They learn that they can use the gesture, using specific language, of removing a blue pawn from each side and still keep the system in balance. We term this physical action a "legal move." The student uses both hands *simultaneously* to remove a pawn from each side, thereby kinesthetically reinforcing the subtraction property of equality.

Recent research by Cook, Mitchell, and Goldin-Meadow (2008) has shown that gestures and bodily movement can play a valuable role in the learning and retention of mathematical concepts.

To illustrate: To solve the equation $4x + 3 = 3x + 9$, we begin with the setup shown below:

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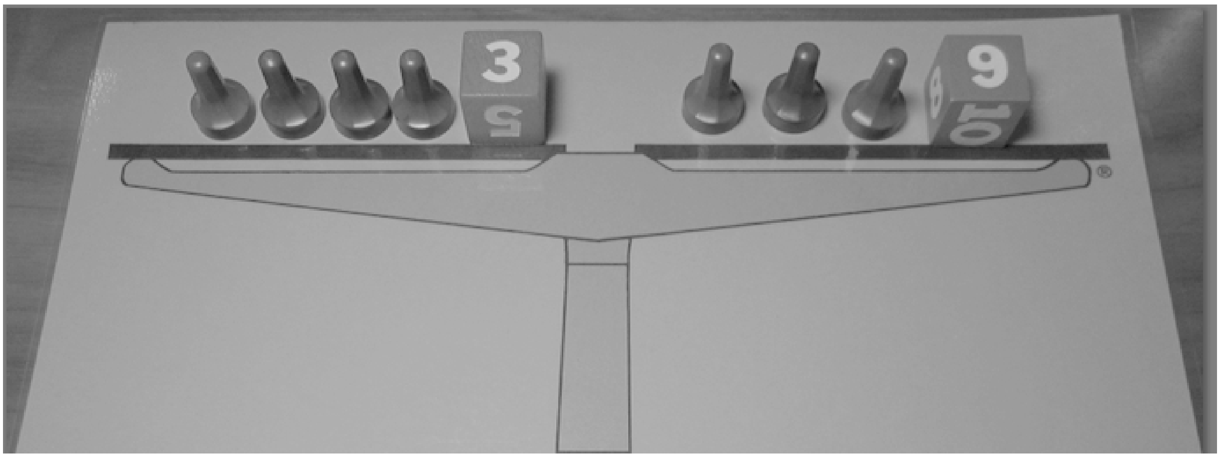
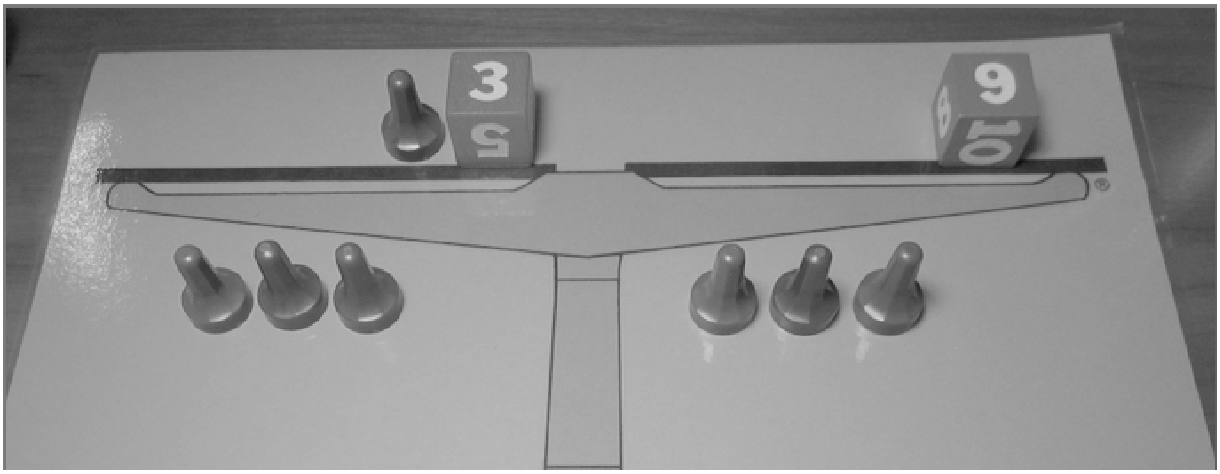


Then the student removes three blue pawns from each side of the equation and ends up with the much simpler setup:



Now the student can see that the value of the blue pawn must be 6, because 6 + 3 = 9. A "check" carried out in the original setup shows that both sides have a value of 27. The student concludes by writing the solution as $x = 6$ and the check as 27 = 27.

This approach to representing, solving, and checking linear equations with the unknown on both sides is easily accessible to students in grades three to nine, including those with limited math skills. Vlassis (2002), working with low achieving eighth graders, noted that "the balance model is especially suited to the study of how to solve equations" (355).

Hands-On Equations is an *algebraic learning environment* in the sense that students often are able to develop some abstract concepts on their own. For example, in one of the lessons students are given the problem $2(x + 3) = x + 10$. They are told that the 2 outside the parentheses tells how many times to set up what is inside the parentheses. Therefore, students set up the problem as:

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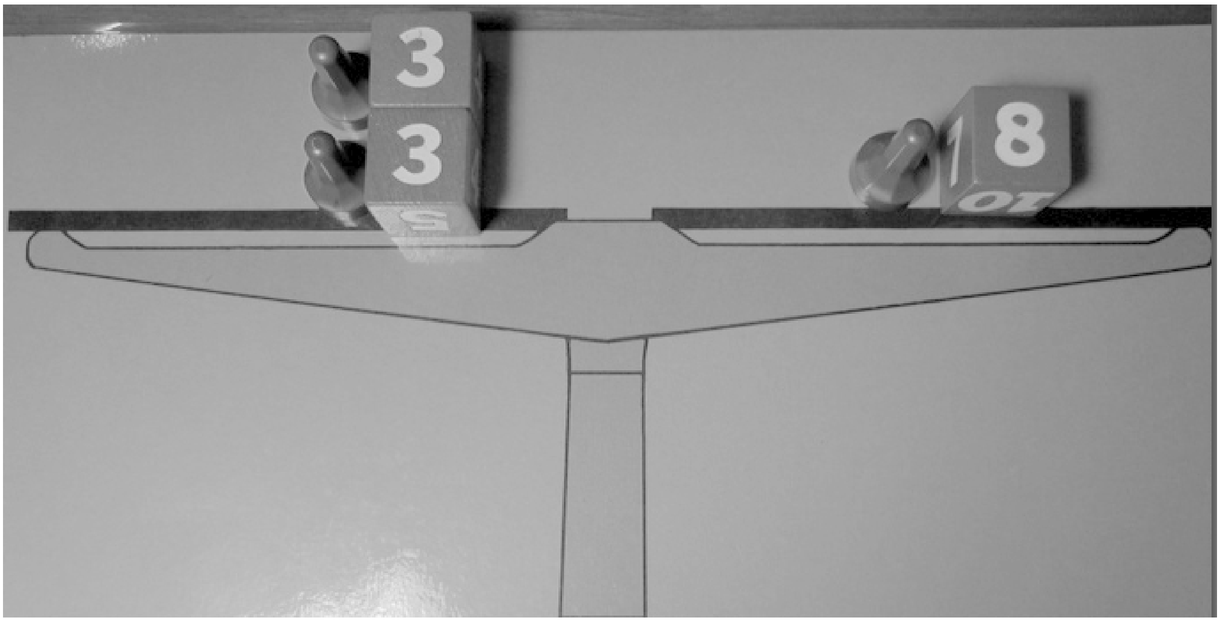
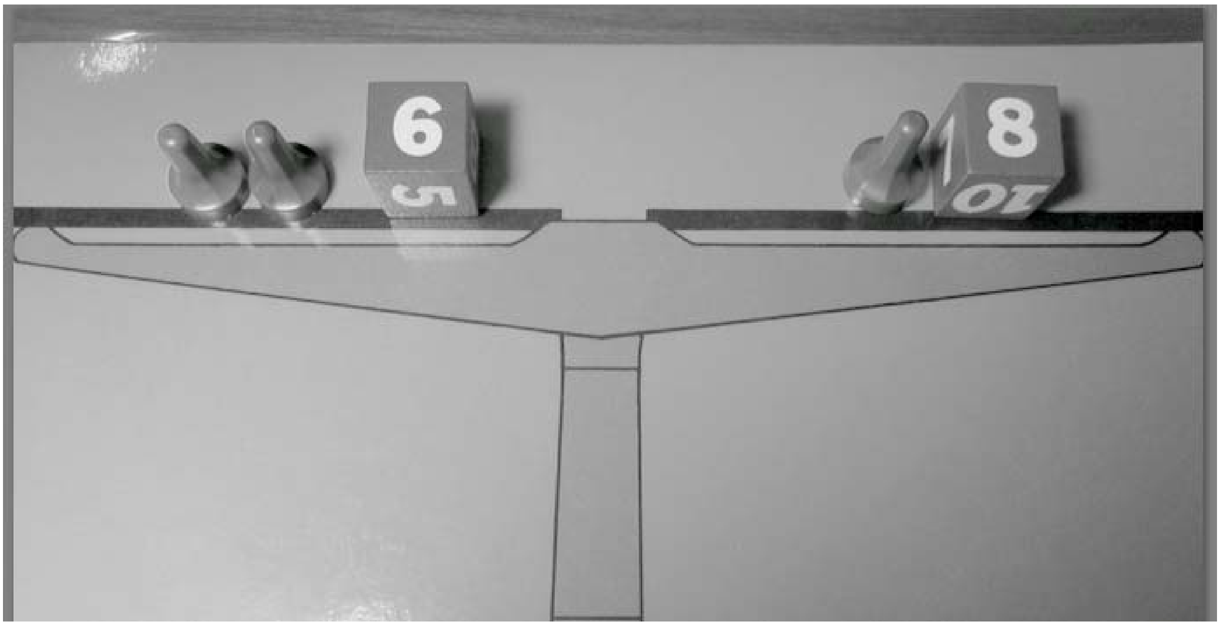


Some students, after doing a few similar problems, see the relationship on the left side and without instruction begin to set up the problem in this manner:



This setup shows that they have discovered the distributive property. It is the simplicity of the Hands-On Equations representation that enables students to discover this and other mathematical concepts.

The New Symbol $\cancel{x}$. Another strategy we use introduces the white pawn, which we call "star" and symbolize as $\cancel{x}$. We tell students that star is the *opposite of x* and may be positive or negative,

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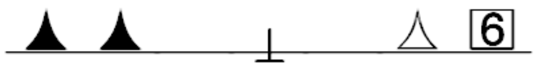
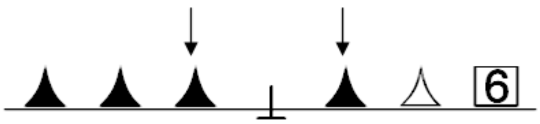
depending on the problem. Later in the program, students learn the conventional symbol: (-*x*). As Brizuela and Schliemann (2004) note, “Students need to be introduced to mathematical notation in a way that makes sense to them” (34).

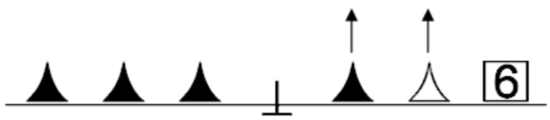
For example, let’s look at the equation $2x = \cancel{x} + 6$, which we can symbolize as:



We need to isolate the 6-cube in a manner that is meaningful to students. One approach that students learn involves the addition property of equality. The student adds a blue pawn to each side, again, using both hands simultaneously:



From this point, the student can remove a pair of opposites from the right side (using one hand), to obtain $x = 2$.



Star, being the opposite, is worth negative 2. We write this as $\cancel{x} = -2$. The check in the original setup is 4 = 4.

Through examples like this, students become familiar with the property of additive inverses and the additive identity property—even if the teacher does not use these terms. The important thing to remember is that students are using principles they understand in any sequence that seems natural to them. They are not memorizing procedures.

In conclusion, it is no longer true that the introduction of algebraic concepts needs to be done abstractly, nor does it need to be “an unmitigated disaster for most students,” as the National Research Council noted. The visual-kinesthetic approach of Hands-On Equations can enable all students to experience success with algebra, starting at an early age.

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