Hands-On Equations Research, Interim Report Nov. 19, 2007
A Comparison of Algebra Achievement by 4th, 6th and 8th graders

By Henry Borenson and Larry W. Barber

Hands-On Equations (HOE), developed by Dr. Henry Borenson, uses numbered-cubes to represent the constants, and blue pawns to represent the variable x. It also uses a scale representation on which the students "set up" the equation. The students then proceed to use "legal moves," which are the mathematical counterpart of the abstract algebraic methods which are used to solve these linear equations. The system thus makes abstract linear equations visual and understandable, and further provides students with the means of solution through a kinesthetic approach which makes sense to them.

The program is unique in that the abstract knowledge base needed by students to solve these equations is transformed into an easily understood and manageable set of verbal, visual, and kinesthetic responses using manipulatives. The program teaches algebraic principles which students in grade 3 to 8 can apply in any sequence desired to solve the given equation. Hence, the students using Hands-On Equations need not memorize a series of steps to solve an equation, as is the case in more traditional instruction. Rather they feel empowered to use their thinking and understanding of basic principles to solve the problem at hand.

This research study, as well as the series of studies of which this is a part, uses a multi-site replications design and a meta analysis procedure to study the effect of the HOE program on many groups of students with different characteristics (regular education students, special education students, elementary, middle, and high school students, inner city, rural, suburban, gifted and handicapped). All of these groups of students will be studied separately. Presently we have data on more than 85 classrooms.

This particular report is an analysis of the results obtained in studies 59a, 102b and 105a. Each of these studies was designed to measure the effects of the first 7 lessons of the HOE program on the learning of algebra by 4th, 6th and 8th grade students, respectively, in the regular education classroom. In addition, each of these studies was designed to determine if there was any significant difference in student achievement by taking the post-test with the game pieces vs, taking the post-test without the game pieces.

A pre-test was given to the students before they were exposed to the program. At the conclusion of Lesson #6, the students were provided with a post-test in which they were at liberty to use their game pieces (the pawns, cubes, and laminated scale). The students were then instructed in Lesson #7, and given a second post-test. This time the students were to take the post-test without using the game pieces. The students however were free to use the pictorial notation they had learned in Lesson #7.

The teachers in this study had been taught the methods of instruction to use with HOE by various Borenson and Associates, Inc. instructors in a one-day workshop sometime in the spring of 2007. The teachers administered the pre-test to their students shortly after the time of their training. They then taught the first six lessons and administered the post test after Lesson #6. The teachers taught Lesson #7 shortly thereafter and then administered the post-test after Lesson #7 a day or two later. Each of the tests consisted of six questions. The students were allowed 15 minutes to take each test.

RESULTS

These three studies involved a total of 22 classrooms containing a total of 418 students: Study 59a involved six classrooms containing 123 4th graders; Study 102b involved eleven classrooms containing 190 6th graders; and Study 105a involved five classrooms containing 105 8th graders.
In each instance, each classroom's data was analyzed independently to provide feedback to each teacher about the performance of their students. Statistical t tests were conducted between the means of the pre-test and the post-test after Lesson #6, between the means of the pre-test and the post-test after Lesson #7, and between the means of the Lesson #6 and Lesson #7 post-tests.

Combined Class Results, Including Mean Scores and T-Values

For each of the combined groups consisting of six 4th grade classes, eleven 6th grade classes, and five 8th classes, the effect size between the pre-test and post-test after Lesson #6, and between the pre-test and post-test after Lesson #7 was large and highly significant. The table below summarizes the result:

Study #	Grade and n value	Class Composition	Pre-test Score (P)	Post-test score Lesson #6 (P6)	t-value (P, P6)	Post-test score Lesson #7 (P7)	t-value (P, P7)
59a	**4th grade, n=123**	102R, 21LD	**1.81**	**5.04**	**22.62**	**5.32**	**29.70**
102b	**6th grade, n=190**	175R, 5ELL, 3LD, 7GT	**2.89**	**5.54**	**25.15**	**5.64**	**22.48**
105a	**8th grade, n=105**	92R, 10LD, 3GT	**3.89**	**5.26**	**8.895**	**5.34**	**9.99**

R= regular students **ELL**= English Language Learners **LD** = Learning Disability **GT**= Gifted and Talented

Additionally, neither the 6th grade nor 8th grade group showed a significant statistical difference in comparing the post-test following Lesson #7 with the post-test following Lesson #6. The 4th grade group showed a significant, but small increase. No group showed a loss in achievement, significant or otherwise, in comparing the results of the post-test following Lesson #7 with the post-test following Lesson #6.

CONCLUSIONS

These three studies demonstrate that 1) Each of the combined group of 123 4th graders, 190 6th graders, and 105 8th grade students achieved a large and significant gain from the pre-test to the post-test following Lesson #6, and 2) This significant gain was maintained on the post-test following Lesson #7, where the students did not use the game pieces (rather, they used the pictorial notation learned in Lesson #7). These results demonstrate that students who learn the HOE methods of solving equations can be equally successful with or without the game pieces. In other words, the students are able to transfer their hands-on learning to the pictorial method presented in Lesson #7, which uses only paper and pencil, and be equally successful in solving the equations.

Looking at the above combined group results again, using percentages, we note the consistency in the scores on both post-tests for each of the three groups:

	Pre-test	Post-test after Lesson #6	Post-test after Lesson #7
Grade 4, n=123	**30%**	**84%**	**88%**
Grade 6, n=190	**48.2%**	**92%**	**93%**
Grade 8, n=105	**64.8%**	**87.7%**	**88.8%**

We make the following observations: 1) HOE seems to be grade-blind, i.e., students at either the 4th, 6th or 8th grade will do equally well with the program. Additionally, whatever inability the program has to yield student scores of 100% applies equally across the various grade levels. 2) We note the gradual increase in the pre-test scores going up from the 4th to the 6th to the 8th grade. It is reasonable to assume that this difference is due to the regular mathematical instructional content that the students had in the intervening two years (for the 6^{th} graders) and four years (for the 8^{th} graders). 3) We note that the post-test score following Lesson #7 for the 4^{th} graders exceeded the pre-test scores for the 8^{th} graders. It is reasonable to inquire whether the first seven lessons of HOE provides a higher level of competence on these particular algebraic concepts and skills than the regular math curriculum does in the intervening two years (for the 6^{th} graders) or four years (for the 8^{th} graders). In order to explore these questions further, we intend to carry out the above study with a larger group of 8^{th} graders, and also to conduct the same study with 7^{th} graders.

Several very important questions arise from the above research: Is it possible that 4^{th} graders, exposed to seven lessons of HOE, can achieve at a higher level than 8^{th} graders (who have not had HOE) on the basic algebraic concepts tested in this study? If this result is confirmed with larger numbers of students, is the critical factor that these concepts are not presented in the regular math curriculum? Or, is it that they are presented but the traditional methods of instruction do not compare in their effectiveness to the methods used in HOE?

Additionally, since the above study suggests that 4^{th} graders do as well as 6^{th} and 8^{th} graders on these algebraic concepts (when presented via Hands-On Equations), it is clear that no purpose is served in holding off instruction on these concepts until the 6^{th} or 8^{th} grade. Hence, the concepts tested in this study, many of which have been traditionally taught at the 8^{th} or 9^{th} grade, can be presented to students as early as the 4^{th} grade, via HOE, with an expectation for a high level of success.

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TEST QUESTIONS FOR STUDY #59a, 102B and 105a

Pre-Test Questions	Post-Test After Lesson #7
1. $2x = 8$	1. $2x = 6$
2. $x + 3 = 8$	2. $x + 3 = 10$
3. $2x + 1 = 13$	3. $2x + 1 = 7$
4. $3x = x + 12$	4. $3x = x + 2$
5. $4x + 3 = 3x + 6$	5. $4x + 3 = 3x + 7$
6. $2(2x + 1) = 2x + 6$	6. $2(2x + 1) = 2x + 10$
Post -Test after Lesson #6	
1. $2x = 10$	
2. $x + 3 = 8$	
3. $2x + 2 = 10$	
4. $3x = x + 4$	
5. $4x + 3 = 3x + 9$	
6. $2(2x + 1) = 2x + 8$	

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