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A Sense-Making Approach to 2/3 x = 4/5

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The 6th grade Common Core State Standards for Mathematical Practice (SMP) expects students to be able to solve fractional linear equations such as *px* = *q*, where *p, x,* and *q* are all specific non-negative rational numbers, e.g., 2/3 *x* = 4/5.

Traditionally, students are taught to solve this type of problem by multiplying both sides of the equation by the reciprocal of the coefficient of x. That solution, requires students to know how to multiply a fraction by a fraction. I would like to present a fraction sense approach to solving this type of problem, an approach that can be used by students who have not learned fraction multiplication. This approach involves first solving for the unit fraction of the unknown, i.e., 1/3 x in the above example, and then using that information to find the unknown.

The Meaning of Fraction Sense

First, I would like to clarify the term "fraction sense." By this term I mean that students work with fractions based on their understanding of the meaning of fractions, rather than mechanically using rules about fraction computation. The benefit of working with fractions using fractions sense is that students have a way to solve problems even if they have forgotten what rule to use, or are not sure if they have remembered the rule correctly. In addition, the student gains the satisfaction that comes from understanding.

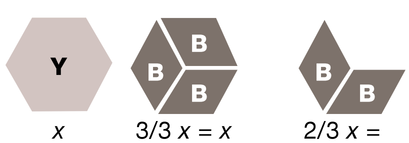
Missing Algebra Standard

Before we consider the problem above, 2/3 x = 4/5, which the SMP presents within the 6th grade standards, I note what to me appears as a glaring omission in the algebra sequence presented in the standards. For, it seems to me, that a prior standard, in the earlier grades, would be to have students solve equations such as 2/3 x = 4, where the fractional part of the unknown is equal to a whole number, rather than equal to another fraction.

Solving 2/3 *x* = 4 using pattern blocks

So, let's begin by considering the problem 2/3 x = 4. The first step is to understand the meaning of the problem: If 2 copies of 1/3 x is 4, how much is x?

We can represent the number x by the yellow block [**Y**]. Since 3 of the blue blocks [**B**] are the same size as the yellow block, each blue block represents the unit fraction 1/3 x and 2/3 x can be represented by 2 blue blocks, which we know have the value of 4 in this example:

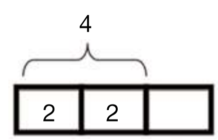


Since both blue blocks together have a value of 4, each blue block is 2. The yellow block consisting of 3 blue blocks, has the value of 6. Hence, the answer to 2/3 x = 4 is x = 6.

Notice that we first solved for the unit fraction 1/3 x and then multiplied by 3 to determine the value of x, since 3 x 1/3 x = x.

Solving 2/3 *x* = 4 using rectangular grids

We can also represent the number, x, by the total area of the 3 equal size boxes shown below. Two-thirds of x would then be the area of two of those boxes, which is given as 4.



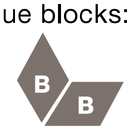
Hence each box, representing the unit fraction 1/3 x, would have a value of 2, and the three boxes, which make up x, have the value of 6. Hence, x = 6.

Let's now apply our learning to solving the fractional equation 3/5 x = 12. We reason as follows: We let x be represented by 5 boxes of equal size. Since 3 of those 5 equal boxes have a value of 12, each one has a value of 4. Since x consists of 5 boxes, x is 5 times 4 or 20. Hence, x = 20.

We can also reason out the solution to 3/5 x = 12 more abstractly, as follows: 3 copies of 1/5 x are 12. Hence, 1 copy of 1/5 x is 4. Since x consists of 5 copies of 1/5 x, x is 5 times 4 or 20.

Solving 2/3 *x* = 4/5 using pattern blocks

We are now ready to look at 2/3 x = 4/5. If we consider the yellow block as having the value of x, 2/3 x would be represented by 2 blue blocks:



2/3 x = 4/5

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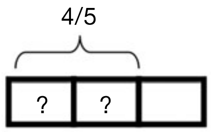
Since both blue blocks together are 4/5, each one is 2/5 (since 2/5 + 2/5 = 4/5). Since 3 of the blue blocks make up *x*, we have *x* = 3 x 2/5 = 2/5 + 2/5 + 2/5 = 6/5. Alternatively, we can add 2/5 to 4/5. Hence, *x* = 6/5.

Solving 2/3 *x* = 4/5 using rectangular grids

We can also let three equal boxes represent *x*. Two-thirds of *x* would be represented by 2 of the boxes, which we know have the combined value of 4/5.

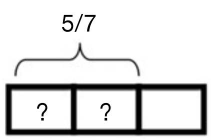


Hence, each box is 2/5. However, *x* is made up of 3 boxes. Hence, *x* = 3 x 2/5 = 6/5.

Let's now apply our learning to solving the fractional equation 3/5 *x* = 12/7. Let's visualize 5 boxes of equal size to represent *x*. Since 3 of those boxes are 12/7, each box is 4/7. Hence, *x*, which consists of 5 of those boxes is 5 x 4/7, which is equal to 20/7. (We can also express this more abstractly: since 3 copies of 1/5 *x* are 12/7, 1 copy of 1/5 *x* is 4/7. Hence, *x*, which consists of 5 copies of 1/5 *x*, would be 5 x 4/7 or 20/7.)

Solving 2/3 *x* = 5/7, where the 2nd numerator is not a multiple of the first numerator

We are now ready to look at 2/3 *x* = 5/7. In this example, two of the 3 equal boxes have a value of 5/7. We need to find the value of one of the boxes. Our question, then, is how much is half of 5/7?



APPROACH 1: Half of 5/7 is (2 1/2)/7 which is equal to 5/14. (Hence, to find *x*, we multiply 5/14 by 3 to give us 21/14.) This approach introduces students to the notation for complex fractions and utilizes their knowledge from grade 4 that we can form an equivalent fraction by multiplying the numerator and denominator of a fraction by the same number, in this case, the number 2.

APPROACH 2: We know immediately that half of 5/7 is 5/14, since doubling the denominator of a fraction results in a fraction half the size of the given fraction. For example, 1/8 is half the size of 1/4 since 8 parts of size 1/8 are needed to equal 4 parts of size 1/4. Likewise, multiplying the denominator of a fraction by 3, results in a fraction one-third the size of the given fraction.

Let's now apply our learning to the equation 3/5 *x* = 7/8. We can visualize 5 boxes of equal size to make up *x*. Since 3 of those boxes are 7/8, one of the boxes would be a third of 7/8, which we know from the above discussion is 7/24. Hence, *x*, consisting of 5 boxes, is 35/24.

Summary

By using fraction sense we are able to solve equations such as 2/3 *x* = 4/5 in a meaningful manner. We simply solve first for the unit fraction 1/3 *x* and then triple the result. This approach can be used prior to the 6th grade by students who know how to add fractions with the same denominator, and who know that 3 copies of the unit fraction 1/3 *x* are equal to *x*.

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